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ABSTRACT

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Beyond word-problems and textbooks: Using reading generatively in the mathematics classroom

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Abstract

The purpose of this monograph is to articulate what it means to "read rich mathematical texts generatively" and the implications such experiences might have for mathematics instruction, based on a descriptive study of selected episodes developed in four secondary mathematics classrooms. These "reading to learn mathematics" (RLM) episodes were the result of action research developed by a collaborative team of mathematics teachers and researchers in mathematics education and reading education; each episode was characterized by the goal of creating opportunities for students to learn about the nature of mathematics, strategies for meaningful learning, and/or technical content by reading math-related text such as historical, philosophical or applications essays and articles using strategies informed by transactional reading theory.

The nature and implications of "reading rich mathematical texts generatively" is developed throughout the monograph in a number of complementary ways. First we articulate the views of mathematics instruction and reading, respectively, that inform the proposed approach to reading mathematics. This approach is then illustrated in practice by means of a "thick description" of two classroom vignettes, which are further examined in light of both transactional reading theory and how these experiences contributed to achieving the new vision for school mathematics promoted by current reform movements. This in-depth analysis of some prototypical examples is complemented by an analysis of all the 18 RLM episodes developed in the four classrooms so as to address the following research questions: (a) what learning opportunities these reading experiences offered to mathematics students; (b) *what*, *why*, and *how* students read in these occasions; and (c) how such reading experiences were orchestrated by the classroom teachers.

The theoretical argument and empirical findings reported in this monograph suggest that, though not the only valuable way to capitalize on reading in mathematics instruction, "reading rich mathematical texts generatively" can provide mathematics teachers with a novel and powerful instructional strategy with which to support the reform of mathematics instruction currently called for by many constituencies.

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1. Introduction

At a time when mathematics educators have come to regard writing (Borasi & Rose, 1989; Connolly & Vilardi, 1989; Countryman, 1992) and talking (Pimm, 1987; Richards, 1991) as valuable ways for learners to recast technical content in their own words, express their feelings about mathematics, reflect on the process of doing mathematics, and puzzle their way through the ambiguity and non-linearity so characteristic of mathematical thinking and knowing, the literature on "reading mathematics" remains mostly focused on instructional strategies that help students comprehend word problems and learn the technical vocabulary of mathematics textbooks (Siegel, Borasi, & Smith, 1989). However much we might agree that students may need this kind of instructional support, this interpretation of "reading mathematics" limits the contribution that reading can make to mathematics instruction as it makes a number of assumptions about *what*, *how*, and, most importantly, *why* students may read in a mathematics class that are at odds with constructivist views of learning and teaching (Davis, Maher & Noddings, 1990; Grouws, 1992; von Glasersfeld, 1991) and humanistic conceptions of mathematics as a discipline (White, 1993).

The first of these assumptions is that the texts most relevant to learning mathematics are pages of well-formed word problems and perhaps some technical definitions and explanations presented in textbooks. Although there is new interest in using children's literature in elementary mathematics instruction (e.g., Griffiths & Clyne, 1991; Whitin & Wilde, 1992), the continued dominance of worksheets and textbooks in mathematics classrooms reinforces the idea that mathematics is a set of ready-made techniques and procedures to be learned. Assumptions about *how* students read mathematics are as deeply embedded in traditional interpretations of "reading mathematics" as assumptions about *what* is read. The "mining" metaphor for reading (Pimm, 1987), in particular, assumes that reading is a matter of extracting meaning from text, a view that directs teachers' and students' attention to features of the text without considering the active role readers play in making sense of such texts. Indeed, the bulk of the research literature on "reading mathematics" has focused on the development of strategies that help students deal with those aspects of technical mathematics texts that present difficulties for learners (e.g., Earp & Tanner, 1980; Moyer et al., 1984; Skrykpa, 1979). Finally, we must consider the assumptions about *why* texts are read that are implicit in the literature on "reading mathematics." On the whole, the texts and reading strategies that have been investigated serve the goals of a "techniques curriculum" (Bishop, 1988) rather than goals such as those articulated in recent reports, which highlight understanding and reasoning as well as beliefs and attitudes associated with successful mathematics learning (e.g., NRC, 1989; NCTM, 1989, 1991). In other words, reading strategies have been introduced in order to help students accurately solve word problems and understand the portions of their textbooks devoted to definitions and explanations, rather than to gain a sense of the historical context or contemporary uses for such techniques, or to learn how mathematicians might have approached a particular problem, or even to set the stage for their own mathematical inquiries.

By juxtaposing the new goals for learning mathematics advocated in recent reports (e.g., NRC, 1989; NCTM, 1989, 1991) and the assumptions underlying most research on reading mathematics, it becomes evident that mathematics educators have tended to underestimate the contribution that reading can make to learning mathematics. If, on the other hand, mathematics educators take advantage of the theoretical shift toward generative and collaborative meaning-making that has occurred within the field of reading, new ways of using text and reading to support mathematical learning become possible. By way of example, let us suppose that one of the goals of mathematics education was to help learners see that mathematics is a product of human activity--value-laden, historically situated, ill-structured, and ever open to change. Because textbooks typically present a truncated image of mathematics, students fail to see that definitions and procedures are the product of a conversation among mathematicians circumscribed by particular values and historical conditions. Teachers might wish to challenge this image by inviting students to read a variety of "rich" mathematical texts (e.g., essays on the history and philosophy of mathematics, mathematical stories and novels, real-life applications of mathematics, accounts of strategies used to frame and solve problems, biographies) that convey something of the complexity of knowing and doing mathematics (Borasi & Brown, 1985). It would be important to understand, however, that such insights would not be automatic but would require some effort on the part of the reader, as recent theories of reading (e.g., Anderson & Pearson, 1984; Goodman, 1994; Rosenblatt, 1978, 1994) and research on comprehension instruction (e.g., Pearson & Fielding, 1991) suggest. Mathematics teachers would therefore want to become familiar with reading strategies that invite readers to act on and transform texts (e.g., by stopping and talking with others as they read, by drawing, by writing down important ideas and questions while reading) in order to fully tap the potential of reading "rich" mathematical texts such as those identified earlier.

This monograph reports the findings of a study that investigated the instructional possibilities of incorporating "rich" mathematical texts such as those described earlier and generative reading strategies informed by transactional reading theory (Rosenblatt 1938, 1978, 1994) into secondary mathematics instruction. The purpose of the monograph is to show what it means to read "rich" mathematical texts generatively and to explore the potential of such reading experiences for mathematics instruction by drawing on data generated in an interdisciplinary research project organized as a collaboration among a reading educator, two mathematics educators, and a group of secondary mathematics teachers--the "Reading to Learn Mathematics for Critical Thinking" project (hereafter RLM project).

More specifically, we begin in Chapter 2 by making explicit the goals for learning mathematics and the theories of reading that inform our work in order to articulate the theoretical foundations of "reading rich math texts generatively." In addition to positioning ourselves with respect to the discourses in the fields of mathematics education and reading education, we believe that our review of literature on reading theory and instruction can contribute to the mathematics education literature as this body of work has been largely ignored yet offers insights into instructional practices that are relevant to learning mathematics.

A description of the overall RLM project, as well as information about how we went about data collection and analysis in the specific study reported in this monograph, will then follow in Chapter 3. We have chosen to go into considerable detail about the research design of this study because we believe that the mathematics education research community can benefit from making public the specific data collection and analysis methods, as well as the thinking and decisions, that go into designing qualitative research studies such as ours.

These theoretical and methodological discussions will be followed by the centerpiece of this monograph, consisting of a "thick description" (Geertz, 1973) of two classroom vignettes in which students used a number of generative reading strategies to make sense and learn with and through a variety of math-related texts. These vignettes, constructed from data produced through collaborative action research, will serve different purposes throughout the monograph. First of all, the detailed narrative account of the vignettes themselves reported in Chapter 4 is intended to offer a holistic portrait of "reading rich math texts generatively" in practice. In the following chapter, these vignettes are then examined in-depth from a transactional reading theory perspective. This discussion will show how reading theory explains the interrelated and generative nature of the instruction described in the vignettes and will begin to identify instructional issues that seem crucial to the effectiveness of this approach in classrooms. The prototypical examples of "reading rich math texts generatively" provided by these vignettes also serve as the basis for a detailed and systematic analysis of how teachers adapted and orchestrated these as well as similar instructional episodes involving reading, and what students gained from these experiences, reported in the next three chapters.

More specifically, in Chapter 6 we will first examine what students gained from engaging in the two episodes reported in the vignettes, and then use the categories generated from this analysis to document the kind of learning opportunities offered by the other 16 "reading to learn mathematics" episodes (RLM episodes hereafter) developed throughout the four secondary mathematics classes involved in the study (see Appendix for a brief description of these instructional episodes). In Chapter 7 we will address the questions of what, why, and how students read in all 18 RLM episodes, so as to identify and discuss complementary ways in which "reading rich math texts generatively" can be effectively employed in mathematics classrooms. In Chapter 8 we will instead analyze how the four classroom teachers orchestrated such reading experiences so as to identify instructional factors critical to the effectiveness of this approach.

Finally, we conclude the monograph by summarizing the findings reported in Chapters 4 through 8, pointing out what is "generative" about the proposed approach to reading mathematics and calling attention to the kinds of support students and teachers will need to tap into the power of reading mathematics generatively identified in our study. As a whole, we believe that the combination of theoretical arguments and empirical findings reported in this monograph can contribute to a better understanding of how reading can be effectively employed as a "mode of learning" in mathematics classrooms - as it suggests a rich set of texts, strategies, and reading purposes that mathematics teachers could capitalize upon for their lessons, sheds light on various elements teachers need to consider to fully tap the learning potential of these kinds of reading experiences, and, most importantly, documents an impressive array of learning opportunities that "reading rich math texts generatively" can offer mathematics students. While we are aware that this is not the only valuable way of capitalizing on reading in mathematics instruction, on the basis of this study we suggest that "reading rich mathematical texts generatively" can provide mathematics teachers with a novel and powerful instructional strategy with which to support the reform of mathematics instruction currently called for by many constituencies.

2. Theoretical Framework

2.1. Reconceived Goals for School Mathematics

At the core of the approach to "reading mathematics" discussed in this article is a set of goals for school mathematics that are crucial to learning and using mathematics today. To understand why we believe that reading rich mathematical texts in a generative mode can make a powerful contribution to students' learning, we need to make these instructional goals explicit and explain why we value these goals over others.

Despite much rhetoric to the contrary, most mathematics instruction today can still be characterized as a curriculum of "techniques" which equates learning mathematics with the acquisition of a limited set of facts and algorithms for doing arithmetic, algebra, geometry, and eventually calculus (Bishop, 1988). Defining the goals for school mathematics in this way means accepting particular views of knowledge and learning, in this case, views associated with positivism and behaviorism. From these perspectives, mathematical knowledge is regarded as a body of established facts and techniques that is hierarchically organized, context- and value-free, and easily broken down into separate skills, and, learning is thought of as the acquisition of these bits of knowledge through watching, listening, memorizing and practicing (Borasi & Siegel, 1994). However widespread these views might be, scholars working from a variety of perspectives have seriously challenged both the theoretical underpinnings and the instructional practices common to a "techniques" curriculum. The most significant of these critiques include an analysis of the kind of mathematical expertise needed in a sophisticated technological world (e.g., NRC, 1989; NCTM, 1989), arguments about the nature of mathematical thinking and knowing (e.g., Borasi, 1992; Ernest, 1991; Lampert, 1990; Schoenfeld, 1992), and constructivist theories of learning, which have found empirical support in research on students' mathematical learning and problem solving (e.g., Davis, Maher & Nodding, 1990; Ginsberg, 1989; Steffe et al., 1983; von Glasersfeld, 1991). Although these critiques differ in some important ways, they suggest a new direction for the goals of school mathematics, one which involves rethinking the mathematical content to be learned, an emphasis on the processes involved in learning mathematics, and attention to students' conceptions and beliefs about the nature of mathematics as a discipline. In the remainder of this section, we will briefly consider each of these goals.

First of all, there seems to be a consensus about the need to rethink the *mathematical content* that students should "learn." This point is especially evident in the curriculum recommendations described in the *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989), and is also supported by the instructional implications of much of recent research in mathematics education (see, for example, the reviews reported in Grouws, 1992). In contrast to the focus on arithmetic, algebra, and eventually calculus, characteristic of the traditional curriculum, the importance of knowing about branches of mathematics such as probability and statistics, number sense and estimation, and patterns and functions is stressed. Furthermore, teachers have been encouraged to pay more attention to students' understanding of the "big ideas" within these branches of mathematics (Steen, 1990) as well as more traditional school mathematics topics. In the context of teaching algebra, for example, this would mean focusing on understanding what an equation is rather than on techniques for solving different kinds of equations; in geometry, it would mean paying attention to ideas such as "shape" and "dimension" rather than memorizing a lot of vocabulary and properties of specific figures out of context. Finally, mathematics educators have pointed out the need for curricula that help students appreciate the connections among the mathematics topics noted above as well as connections to other fields that use mathematics (NCTM, 1989).

As part of this rethinking of the school mathematics curriculum, we have also seen a new emphasis on instruction that explicitly supports the *process* of learning and doing mathematics. It is significant, for example, that the NCTM Standards articulate the key goals for school mathematics in terms of processes (solving mathematical problems, reasoning and communicating mathematically) and attitudes (valuing mathematics, gaining confidence in one's ability to do mathematics) above learning specific content or basic skills (NCTM, 1989, p.5). Support for this emphasis on process can be found in the research literature on problem solving. This body of work has attempted to identify the kind of knowledge and strategies needed to be a successful problem solver and to develop instructional practices that help students learn to use such strategies (e.g., Charles & Silver, 1988; Schoenfeld, 1985, 1987). Some of this work has focused on the importance of metacognition-- awareness of and reflection on the process of framing and solving problems--in successful problem solving (e.g., see review reported in Schoenfeld, 1992). This interest in metacognition, along with other theoretical influences, has contributed to a growing appreciation for the mediating role of language in problem solving and new interest in research on students' use of language and other modes of representation to reason and communicate mathematically (e.g., Connolly & Vilardi, 1989; Janvier, 1987; Richards, 1991). Although research on these topics has been fruitful, more instructional approaches grounded in such research are needed to help students develop learning strategies that will enable them to frame and solve the kind of problems they are likely to encounter in mathematics classes and everyday life.

One aspect of learning mathematics that has received far less attention in the research literature and even in documents such as the NCTM Standards focuses on learning about *the nature of mathematics as a discipline.* Yet, this seems a critical dimension of a student's mathematical education, especially in light of Schoenfeld's contention that "a fundamental component of thinking mathematically is having a mathematical point of view, that is, seeing the world the way mathematicians do" (Schoenfeld, 1992, p. 340). Support for this argument comes from several sources. Social constructivist theories of learning, for example, highlight the social as well as cognitive aspects of learning, and suggest that becoming part of a community of practice is essential to becoming an expert in a discipline (e.g., Lave & Wenger, 1989; Bishop, 1988; Resnick, 1988; Schoenfeld, 1992). As Resnick (1988) has noted, "...becoming a good mathematical problem solver -- becoming a good thinker in any domain -- may be as much a matter of acquiring habits and dispositions of interpretation and sense-making as of acquiring any particular set of skills, strategies or knowledge" (p. 58).

Research on students' beliefs about mathematics also suggests that developing an awareness of the nature of mathematics is a crucial part of learning mathematics. These studies (e.g., Schoenfeld, 1989; Borasi, 1990, 1992; Buerk, 1981, 1985) have revealed a number of widespread beliefs about mathematics that are unfounded and hence dysfunctional for learners. For example, many students believe that:

- there is only one correct way to solve any mathematical problem -- usually the rule the teacher has most recently demonstrated to the class; [...]
- mathematics is a solitary activity, done by individuals in isolation;
- students who have understood the mathematics they have studied will be able to solve any assigned problem in five minutes or less;

- the mathematics learned in school has little or nothing to do with the real world. (Schoenfeld, 1992, p.359)

These beliefs are cause for concern not only because they misrepresent the nature of mathematical thinking, but because they promote unproductive learning strategies and habits of mind. Faced with a problem that involves more than the application of a stated algorithm, students who hold the above mentioned beliefs may think, "It is no good to try to reason things out on your own; [...] staying too much on a problem is a waste of time; [...] history and philosophy of mathematics are irrelevant to learning mathematics" (Borasi, 1992, p. 208). Such dysfunctional ways of thinking thus places students at risk for developing math anxiety (e.g., Borasi, 1990; Buerk, 1981).

In calling for instruction that helps students appreciate the nature of mathematics and develop beliefs and strategies associated with the mathematics community, we should not forget that mathematicians continue to disagree about how to define the "nature" of mathematics. Nor should we forget that previous reform movements also tried to help students understand the nature of mathematics. Recall that this was an explicit goal of several of the "New Math" projects in the 1960's, although it meant familiarizing students with what were thought to be unique and characteristic aspects of mathematical thinking, such as notions of structure and abstraction. Those working from a "humanistic" perspective on mathematics, on the other hand, have suggested a different interpretation of the nature of mathematics, one that would help students see mathematical knowledge as socially, culturally, and politically constructed. Rather than highlight the absolute and authoritarian image of mathematics, humanistic mathematics educators contend that mathematics is a human endeavor as fallible and value-laden as any other (e.g., Borasi, 1992; Brown, 1982; White, 1993). This view of mathematics takes on special significance if educators want students to see mathematics as a way of thinking that has value and meaning for their lives.

The new goals for school mathematics outlined above provided the impetus for creating an approach to reading mathematics that focused on "rich" mathematical texts and generative reading strategies. In particular, we were interested in providing mathematics teachers with instructional strategies that would help students develop an appreciation for the humanistic nature of mathematics, and in doing so, challenge their taken-for-granted beliefs and learning strategies. By going beyond the word problems and textbooks typical of the reading material students encounter in their mathematics classes, and inviting students to instead read brief essays on such topics as the history of mathematics, mathematical applications, and the like, we imagined that students might begin to broaden their understanding of mathematics. At the same time, we expected that, depending on the specific content of the texts read and the learning experiences organized around these texts, a generative reading of "rich" mathematical texts might also encourage students to learn technical content associated with traditional mathematics instruction (e.g., calculating mean, mode, and median; proving congruence of triangles; identifying the properties of specific geometric shapes) in ways that highlighted meaning and context. Finally, we hoped to expand students' repertoire of learning strategies through the use of generative reading strategies. In addition to helping students actively make sense of particular mathematical texts, we assumed the social and generative character of these strategies could support mathematical problem solving, communication, and learning more generally. In the section that follows, we discuss the reading theories these assumptions about "reading to learn mathematics" were based upon and examine several of the specific strategies we used in planning instructional experiences.

2.2 Recent Directions in the Theory and Practice of Reading

Among reading educators, theories of reading are as controversial as questions about the nature of mathematics are for mathematics educators. Though disagreements remain, the last 30 years have witnessed radical changes in reading theory and research as a result of conceptual shifts in the fields of linguistics, cognitive psychology, and literary criticism. The image of reading that has emerged from this work is one in which readers are neither passive in the face of the text, nor blind to the social event of which reading is part. Instead, readers are regarded as active meaning-makers who use their knowledge of language and the world to construct interpretations of texts in light of the particular situations within which they are read. As noted earlier, researchers interested in the problem of "reading mathematics" have generally approached reading as a matter of "mining the text," and yet, this metaphor ignores the conceptual shifts that have had such a powerful influence on reading instruction in both elementary and secondary classrooms. In this section, we briefly summarize the two theoretical positions on reading that framed the research presented here before discussing several instructional strategies researchers have developed to help students become active meaning-makers.

Theoretical Base for a Generative View of Reading

Prior to the publication of Noam Chomsky's Syntactic Structures (1957), reading was thought of primarily as a perceptual process in which the visual information was translated into oral language; from this perspective, comprehension involved little more than listening to the words that were produced as one read (Pearson & Stephens, 1994). Chomsky's critique of behaviorist theories of language and language learning marked the beginning of an interest in reading as a language process, not simply a "decoding" (or, more accurately, "recoding") activity. As early as 1965, Goodman's research on readers' oral reading errors led him to propose a "psycholinguistic" model of the reading process which characterized reading as an active process in which readers engaged in a cycle of sampling, predicting, confirming, and integrating language cues and their background knowledge to construct an understanding of the text (Gollasch, 1982; Goodman, 1967, 1985, 1994). Frank Smith (1971) reached similar conclusions about the nature of the reading process and proposed a definition of comprehension as a matter of getting one's questions answered, thus assigning readers the leading role in the process.

The emphasis on readers as active meaning-makers also became a hallmark of cognitive theories of reading comprehension, which drew on the computer as a metaphor for human cognition (e.g., Adams & Collins, 1979; Anderson & Pearson, 1984; Rumelhart, 1977). Much of this work was grounded in the idea that knowledge is represented in memory as schema, an abstract structure that represents the relationships among its component parts (Rumelhart, 1980). Researchers repeatedly showed that even in cases where readers could recognize all the words, comprehension was not achieved until the relevant schema was brought to bear on the text (e.g., Bransford & Johnson, 1972). The classic example of this is the sentence "The notes were sour because the seams split," which make little sense until the schema for bagpipes is instantiated (Bransford & McCarrell, 1974). Schema-theoretic accounts of reading comprehension suggest that generating inferences based on prior knowledge is central to understanding text as it enables readers to reduce the ambiguity they encounter and experience the "click of comprehension." Yet, this "click of comprehension" is not the same for every reader. Numerous studies (e.g., Anderson, Reynolds, Schallert, & Goetz, 1977) showed that readers' interpretations of a single text varied depending on their personal schema, a finding that makes sense once readers are assumed to be active interpreters and texts are regarded as blueprints rather than containers of meaning. Despite differences in theoretical grounding (theories of knowledge representation rather than language) and methodological approach (experimental designs rather than observations of oral reading of full-length texts followed by retellings), the findings of cognitive theories of reading contributed to the shift toward constructivist theories of reading comprehension first articulated by Goodman and Smith.

A number of criticisms of schema theory, some theoretical and some methodological, have led to new directions in cognitive approaches to reading which have important implications for instructional practice. For example, an important new direction in cognitive theories of reading comprehension is exemplified by Hartman's (1991) research on the intertextual links readers make within and across texts. Arguing that the focus on single texts rather than multiple texts oversimplifies most reading encounters, Hartman highlights the openness and interconnectivity of making meaning with texts. Attempts to understand and support readers as they deal with this kind of complexity represent the new wave in reading research, an effort that has been significantly aided by drawing on literary theories of reading.

Literary theorists also began to assign the reader a more central role in the reading process in the 1970's (e.g., Bleich, 1978; Eco, 1979; Rosenblatt, 1978) as the influence of New Criticism began to wane. New Criticism, an approach to reading literature that dominated and continues to dominate the teaching of literature in high schools and colleges, was based on the assumption that texts contain objective meanings that can be revealed through close readings of the text. Louise Rosenblatt's transactional theory of reading (1938, 1978, 1994) has been especially influential in redirecting reading theory, research, and practice away from an exclusive focus on the text to an interest in the reader's aesthetic or "lived-through" experience and her/his response to that experience. Like schema-theoretic approaches to reading, Rosenblatt treats the reader as an active participant in the event, bringing her/his knowledge of language and background experiences, as well as interests, values, and feelings to the text. There are important differences between these two theories, however, that relate to the kinds of texts read, the nature of the reader/text relationship, and the role of context.

Though Rosenblatt is careful to state that her transactional theory applies to the reading process in general, as a teacher of literature she is most interested in the way readers *experience*, not just "comprehend," literary works and, indeed, she grounded her theory in observations of her students' engagement and response to literature over a 40 year career as a Professor of English Education. This approach to theory construction is quite distinct from that of the cognitive psychologists, but it had the advantage of dealing with complex texts as opposed to the short, artificial texts that schema theorists employed. Equally significant is her characterization of the reader/text relationship. Whereas cognitive theories have portrayed reading as an *interaction* between reading and text in which the two influence one another, Rosenblatt regards the reader/text relationship as "a complex, nonlinear, recursive, self-correcting transaction" (1994, p. 1064). As an ongoing cycle of generative and reflective meaning-making, the reading event begins with a tentative purpose or expectation that guides the reader as she/he encounters the printed text. The text, consisting of printed signs with the capacity to act as symbols, calls up in the reader past experiences, feelings, and images which the reader organizes and synthesizes to create a tentative framework which may be revised, rethought, or rejected as the event proceeds. Reader and text thus become part of an ongoing event in which each element shapes and is shaped by the other. And, because each transaction is a living thing, bound up with a particular personal, social, and cultural matrix, the meanings produced must be thought of as unique to the event, a mix of public and private meanings and not reconstructions of some static text. The expectation, therefore, is that no reader will arrive at the same meaning as another reader, nor will the same reader generate the same interpretation at a different point in time.

Finally, Rosenblatt's theory and schema-theoretic approaches to reading differ in the attention paid to context. By arguing that her theory is not a theory of a generic reader but of particular readers reading in particular situations, Rosenblatt rejects the assumption, implicit in schema theoretic views of reading, that reading can be understood in terms of reader factors and text factors alone. As noted earlier, she sees reader/text transactions as part of a personal, social, and cultural matrix, which suggests that reading events be considered in relation to the purposes, social relations, knowledge, interests, feelings, and values that constitute them. Though Rosenblatt's treatment of context does not amount to a situated view of reading, as characterized by sociolinguists (e.g., Green & Meyer, 1991), it nevertheless opens the door to considering what she calls "broadening circles of context" (Rosenblatt, 1994, p. 1085) when analyzing reading transactions.

Implications for Reading Instruction

Reading educators have used both the cognitive theories of reading and Rosenblatt's transactional theory as a framework for developing and researching instructional strategies that support active, meaningful reading and learning (e.g., Beach & Hynds, 1991; Pearson & Fielding, 1991; Tierney & Pearson, 1981/1994; 1992/1994). The instructional strategies that grew out of the early schema-theoretic research on reading comprehension emphasized the active role of the reader (both in terms of background knowledge and inferencing), but continued to judge a reader's understanding in terms of fidelity to the referential meaning of the text, as defined by the researchers. Over the last 15 years, however, the combined influence of literary and sociolinguistic theories (e.g., Bloome & Green, 1984) has led reading researchers working from a cognitive perspective to conclude that readers make sense of texts through the social construction of interpretations that have meaning in both personal and cultural terms. This shift from text-bound to generative views of comprehension is reflected in the research on comprehension instruction, as Pearson and Fielding (1991) note in the conclusion to their review of this work:

> Students understand and remember ideas better when they have to transform those ideas from one form to another. Apparently it is in this transforming process that *author's* ideas become *reader's* ideas, rendering them more memorable. Examined from the teacher's perspective, what this means is that teachers have many options to choose from when they try to engage students more actively in their own comprehension; summarizing, monitoring, engaging relevant knowledge, creating visual representations, and requiring students to ask their own questions all seem to "generate learning". (p. 847)

The current wave of comprehension instruction research takes this notion of generative meaning-making even further and focuses on the creation of classroom practices that expect reading to involve ambiguity and multiple perspectives and therefore place value on opportunities for dialogue, inquiry, and reflection. Not surprisingly, these practices are marked by a move toward "authentic" texts (i.e., texts written for purposes other than research or teaching) and purposeful instructional experiences (i.e., experiences that go beyond knowledge transmission and display); moreover, these practices reposition learners and teachers such that teachers become facilitators who help learners take control of their reading and learning rather than experts with authoritative meanings to transmit (e.g., Harste & Short, with Burke, 1988; Pearson & Fielding, 1991).

The reading strategies we chose to incorporate into the "reading to learn mathematics" project were grounded in Rosenblatt's transactional theory of reading, yet consistent with the findings of more cognitively-oriented research on comprehension instruction. Each strategy invited readers to act on and transform the text so as to generate and reflect on the meanings they made. The first, "say something" (Harste & Short, with Burke, 1988), is based on the assumption that comprehension results from an evolving dialogue with both the text and other readers. Readers select a partner and a text and talk their way through a text, stopping at reader-designated points to share their confusions, questions, and feelings, make connections to background knowledge and experience, put the text in their own words, or generate hypotheses. Rather than teach students a prescribed way to process the text (such as first summarizing, then formulating a question), the more open-ended approach allows readers to monitor their reading in whatever ways they choose and to reflect on and revise their meanings as the event unfolds. The advantage of this is that students can make public their responses, however tentative, and explore them before continuing the reading; in this way, students learn that comprehension is not automatic but requires some action on their part.

The second strategy, "cloning an author" (Harste & Short, with Burke, 1988), engages the reader in a similar process of reading and reflection. In this case, however, readers are asked to stop reading whenever they choose and write what they regard as important ideas on 3x5 index cards or "post it" notes. After they have finished reading the text or texts in this manner, they are asked to arrange their cards in such a way as to show the relationships among ideas. These "maps" can then be discussed with peers and may be revised in the course of that discussion; the advantage of using 3x5 cards or "post it" notes is that they can be easily rearranged to reflect new connections and relationships. The emphasis on creating a "map" or visual arrangement of their ideas is similar to instructional strategies that focus readers' attention on the structural relations among important ideas in the text, that is, on text structure. The chief difference between "cloning an author" and text structure strategies is signaled in the name of the strategy; rather than suggest that readers clone or reproduce the *author's* text structure, the strategy invites readers to take on or "clone" the activities of an author, that is, the selection and organization of key ideas. Here again, the openness of the strategy serves as a reminder to readers that they must make choices and connections when they read and are not solely bound to the text. The "card" strategy thus teaches students that texts are always ambiguous and reading is a fluid process that involves making decisions about what is important, drawing connections between ideas, and revising those decisions and connections throughout the reading event.

The third strategy we introduced is called "sketch-to-stretch" (hereafter also referred to as "sketching;" Harste & Short, with Burke, 1988; Siegel, 1984, in press), and takes the idea of transforming the text even further by inviting readers to draw their interpretations of the text. After reading a text or texts, readers are asked to "draw what they made of or learned from the text" and then share their sketches with other students. An effective way of sharing sketches is to let students "read" and interpret the sketch before giving the artist a chance to explain her/his drawing; this approach creates further opportunities for students to generate and reflect on their understanding of the text/s under study. The underlying assumption of this strategy is again revealed in the strategy's name; the act of recasting meanings generated in one sign system (language) in another (visual art) allows readers to reflect on their interpretations from a different perspective, a move which can lead readers to new insights.

Two key points should be reemphasized about the reading strategies described above. The first is that these strategies are not intended to serve as prescriptions for what and how to think but as ways for students to experience reading as a generative and reflective activity, one that involves feeling as well as thinking. This means that students not only experience the risk-taking and flexibility that this kind of reading calls for, but have opportunities to express their feelings about the act of reading itself, especially the feelings of confusion and frustration that typically arise when reading challenging texts. Second, these strategies are not viewed as ends in themselves, but heuristics or springboards for dialogue, reflection, and inquiry. Notice that opportunities for readers to share their thinking with others was a common feature of each strategy and reflects the belief that the process of transforming private thoughts into public meanings gives all members of the classroom community a chance to socially construct meaning and, in doing so, push their thinking and inquiry forward.

3. Research Design

3.1. An Overview of the "Reading to Learn Mathematics" Project

The study reported in this monograph is only one component of the larger "Reading to Learn Mathematics for Critical Thinking" project. The RLM project, (supported by a grant from the National Science Foundation - award # MDR-885158) was designed jointly by a mathematics education researcher and a reading education researcher as an exploratory study that sought to create a new synthesis of reading and mathematics based on the integration of rich mathematical texts and transactional reading strategies into mathematics instruction. The overall research question informing the project was articulated in its original proposal (Borasi & Siegel, 1988) as: How can mathematics instruction take advantage of transactional models of reading so as to foster critical thinking, defined as an attitude of inquiry (Siegel & Carey, 1989), and a deeper understanding of mathematics? In addition, several sub-questions were posed: What implications for the design of instructional experiences in the mathematics curriculum can be drawn from research on reading comprehension?; How do students actually use the learning opportunities provided in "reading to learn mathematics" (RLM hereafter) experiences?; and, What do students gain from their use of "reading to learn mathematics" strategies?

Addressing these questions required first of all the development of a rich set of RLM experiences, that is, classroom situations where mathematics students would engage in the reading of rich mathematical texts, using reading strategies grounded in a transactional theory of reading, in order to achieve a better understanding of mathematical content, processes, or characteristics of mathematics as a discipline. Since it was our intention to develop these experiences in collaboration with secondary mathematics teachers, we also realized that some preliminary classroom experiences and professional development initiatives would be necessary. The RLM project was therefore designed to take place in three phases over a two-year period:

Phase 1: Development of an illustrative unit involving RLM experiences in the context of one teacher's classroom;

Phase 2: Professional development seminar for secondary mathematics teachers interested in exploring uses of reading in their teaching;

Phase 3: Collaborative action research with a selection of the teachers who had participated in the seminar and were committed to developing and incorporating some RLM experiences in at least one of their classrooms.

Since the second and third phases of the RLM project are most relevant to the research reported in this article, we will provide some additional information about them in what follows.

The Professional Development Seminar (Phase 2)

Given the fact that the approach to reading mathematics we were proposing would be novel for most secondary mathematics teachers, we thought it crucial to offer a carefully designed professional development seminar for those teachers interested in exploring the use of reading in their mathematics classes. In the semester-long seminar we developed to meet this need, 12 teachers were first of all invited to take the stance of a learner and experience first-hand how *they* could use a variety of texts and strategies to learn about such topics as non-Euclidean geometries and infinity, in the context of units orchestrated by the facilitators (see Borasi & Siegel [1990] and Smith [1994] for a report of these "Alternative Geometries" and "Infinity" RLM experiences, respectively). With these shared experiences as a frame, the group then focused on setting the stage for collaborative action research. In this segment of the seminar, the participating teachers engaged a variety of activities, including: examining the data from the first illustrative RLM experience carried out in Phase 1, which raised many questions about the difficulties of implementing such lessons in traditional mathematics classrooms; looking more closely at their own approaches to "reading mathematics;" beginning to locate relevant reading materials that they could use with their own students; developing some of their own RLM instructional experiences and, in a few cases, trying them out in their classes.

Although this monograph does not explore what the participants took from the professional development seminar, either in the way of specific activities or the shared experience of becoming a member of a "community of inquirers," it is important to keep this question in mind when considering the instructional experiences that were later generated by some of the participating teachers in Phase 3.

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Collaborative Action Research (Phase 3)

In the school year following this professional development seminar, four of the participating teachers (representing varying years of teaching experience and quite different instructional contexts) volunteered to join with the researchers in planning and implementing some innovative units involving reading experiences. Each of these units was designed by a "research team" comprised of the reading researcher, one of the mathematics education researchers, the classroom teacher, and often another teacher who had participated in the seminar and, though unable to develop RLM experiences in his/her classroom that year, wanted to remain involved in the project.

As noted earlier, the classroom research component of the RLM project was designed as collaborative action research in which the members of each research team met regularly to plan and discuss instructional experiences that would create the best learning opportunities possible to achieve the goals set for that unit or course. It is important to state that, although the researchers were primarily interested in the ways that "rich" mathematical texts and generative reading strategies could be incorporated into the instructional unit, they did not limit their concern to those activities. Instead, they joined with the entire team, but especially the classroom teacher, to create purposeful, functional, connected learning experiences for students, even if sometimes it meant not introducing reading in the ways initially planned. The priority of the research team, in other words, was meaningful instruction. Hence, team meetings were usually devoted to thinking about the unit as a whole and the contribution that various texts and reading strategies might make. Collaboration of the kind we attempted is characterized by dialogue and negotiation in light of each teacher's particular situation. As a result, each research team developed a distinctive approach to the classroom research, in most cases working jointly on the development of units which addressed topics that were part of the extant curriculum (e.g., logic, probability, geometry) and in one case working in a more consultative way to support the classroom teacher's own curricular plan (e.g., a semester-long course on mathematical connections). Despite these differences, team discussions tended to focus on revising instructional plans in response to specific problems that arose, and understanding what was happening in the classroom.

The research was conceptualized as action research rather than ethnographic research because the express purpose of the research was to contribute to the educational environment in whatever ways the classroom teachers felt were appropriate. Unlike educational ethnographers, who are interested in developing emic understandings of classroom life or the intersection of school and culture, our focus was improving mathematics instruction through the use of reading. Though some educational ethnographers have argued that ethnographic research might be oriented toward educational change (e.g., Cazden, 1983; Erickson, 1979; Moll & Diaz, 1987), others have suggested that adopting that kind of stance reduces ethnography to a set of methods since it is the construct of culture and not long-term participant-observation alone that makes fieldwork ethnographic (e.g., Heath, 1982; Zaharlick & Green, 1991). What we shared with ethnography, however, was an appreciation for the reflexive, contextual, and emergent nature of the enterprise.

As Carr and Kemmis (1986) have noted in their explication of transformative action research, the research presented in this monograph was "not on or about education, ... [but] in and for education" (p. 156, emphasis in original). In an attempt to avoid the dualisms characteristic of traditional ethnographic research, this research did not separate understanding from application nor the researcher from those "researched" (Gitlin, Siegel, & Boru, 1989). This meant that in addition to documenting the ongoing classroom events in field-notes, the university researchers contributed ideas and insights to make learning meaningful for the students, facilitated small group discussions with students, and, in most of the classrooms, played either a leading or supporting role in the actual teaching. In short, we sought to avoid a purely instrumental and technical approach to the research and to instead treat our "methods" as expressions of our commitments as educators. By aligning purpose and method in this way, we hoped to create something closer to a community of practice (Lave, 1988) than either an ethnographic or a quasi-experimental study would allow.

3.2. Instructional Experiences and Data Generated through Collaborative Action Research

From this collaborative research, four units (lasting from 2 to 6 weeks) and a semester-long course where reading played an important role were developed (note that, for the sake of brevity, hereafter we will refer to all five experiences as "RLM units"):

- *a 6-week unit on geometry,* taught to a class of 21 students enrolled in an 8th grade mathematics course at a rural/suburban school (classroom teacher: Ms. G., first year teacher);

- *a 3-week unit on probability,* taught to 21 students (mostly 9th graders) enrolled in "Course I" (the first of a series of three integrated math courses intended primarily for college bound students in New York State, and covering a curriculum intended to prepare the students for a final state-wide examination) at a suburban high school (classroom teacher: Ms. P., 5 years of teaching experience);
- *a 2-week unit on logic,* taught to 26 students (mostly 9th-10th graders) enrolled in "Course IA" (Part 1 of a two-year sequence intended to cover the curriculum content of "Course I" described earlier, but at a slower pace) at a large urban high school (classroom teacher: Mr. I., 17 years of teaching experience);
- *a 3-week unit on probability,* taught by the same teacher (Mr. I.) in the same Course IA, and designed along the lines of the one developed for Ms. P's class;
- *a semester-long (18 weeks) course called "Math Connections,"* in which 14 students in grades 9-11 from an alternative urban high school participated (classroom teacher: Ms. F., 13 years of teaching experience).

In all these RLM units, both *what* was taught and *how* it was taught presented a considerable departure from traditional mathematics classes. Most importantly, in the units on geometry, logic, and probability, goals such as constructing an in-depth understanding of key concepts within that topic, and developing an appreciation of the historical development and potential applications of that topic were added, and in most cases even given priority over the goal of acquiring some of the "techniques" mentioned in the official course curriculum.

The course on "Math Connections" needs a little more explanation, given its very unusual focus and instructional goals. The overall goal of the course was to broaden students' conceptions of mathematics beyond the commonly held notion that mathematics deals with operations on numbers. The teacher thus designed "Math Connections" as an opportunity for students to engage in problem-solving, reasoning, communicating, and developing an appreciation for mathematics as a humanistic discipline while exploring the connections between mathematics and disciplines such as art, music, literature, and science. Because hearing students' voices was the core value in this teacher's belief system, the course was organized as a series of inquiries into math connections. Students had many opportunities to explore the idea of math connections and reflect on these experiences before defining a focus for their inquiry (they decided to study the math connections of racing) and carrying it out. The racing inquiry, in turn, created opportunities to learn new strategies and raise new questions, which were applied and pursued later in the course. Though this course used reading in a variety of ways, the RLM episodes for this class occurred primarily in the section of the course devoted to learning new strategies for sharing information and making sense of the kinds of texts they had encountered during their study of racing (see episodes C1-C7 in Appendix). The course concluded with individual inquiries generated from the careers and application sections of traditional math textbooks (see episode C8 in Appendix).

Each of the five RLM units described above were documented over the entire duration of the unit, using data collection procedures drawn from the interpretative research tradition (Erickson, 1986; Lincoln & Guba, 1985). At least one of the researchers was present at every class meeting during the RLM unit and recorded observations in field-notes in addition to audio- and/or video-taping the lesson. Photocopies of all students' work and instructional materials were made. In addition, weekly meetings of the research team were audio-taped and the more informal daily meetings that often followed the class were documented in field-notes.

It is worth noting that, although the five units listed above represent the planned follow-up to the professional development seminar, as specified in the original design of the project, two of the teachers continued to develop reading experiences in their classes without the support of the research team. Some of these experiences were partially documented by the teachers themselves and as such constitute interesting examples of teacher-research.

3.3. Data Analysis

The research reported in this monograph examined the data generated by the collaborative action research in Phase 3 of the RLM project so as to better articulate the meaning and instructional implications of "reading rich math texts generatively" in the context of mathematics classes. More specifically, data on the generative reading experiences developed in the four classrooms were analyzed in terms of the following research questions: (1) What, why, and how did students read in these experiences?; (2) How did the classroom teacher orchestrate such reading experiences?; and, (3) What did the students gain from these experiences?

Addressing these questions required a multi-stage process involving the analysis of data produced in the 5 RLM units described above. The first step (which in most cases occurred close to the time the classroom experiences took place) involved the construction of written narratives for each class from field notes and mechanical recordings. These narratives were not verbatim transcripts, but did include a significant amount of transcribed classroom discourse as well as descriptions that attempted to capture the substance, tone, and meaning of ongoing classroom interactions. In light of the recent scholarship on ethnographic writing (Clifford, 1983; Clifford & Marcus, 1986; Tyler, 1985; van Maanen, 1988), it should be noted that these narratives adhered to traditional rhetorical practices in ethnographic writing in that they were written as realistic accounts which offered a single point-of-view. The production of such narratives is clearly not straightforward and represents the intentions and biases of the researcher "flattened" through writing. Still, they allowed the researchers to reduce the data in a way that allowed further analysis and sense-making.

The next step involved the development of a unit of analysis, which would be rich enough to allow us to understand the generative reading experiences without stripping away meaning, and at the same time manageable enough to allow us to analyze patterns in the classroom narratives related to what was read, why it was read, and how it was read. After considerable debate and exploration of a number of alternatives, we settled on a unit of analysis that we called an *RLM episode* and defined it as "the set of learning experiences developed around the generative reading of a rich math text." The four authors (consisting of the mathematics educator and reading educator who initiated the project, another mathematics educator who played the role of research associate in Phase 3 of the project, and one of the collaborating classroom teachers) then engaged in a first analysis of the classroom narratives in order to identify all the RLM episodes developed in the 5 units. More specifically, for each unit, one of these researchers (who had participated in the research team that developed the unit) examined the data collected on that unit and, using the above definition, identified all RLM episodes in it as well as borderline cases. The list thus generated was then discussed and agreed upon by consensus by all four researchers (a group which always included at least one person who did not belong to the research team who had developed the unit).

This process was neither straightforward nor unproblematic, since we encountered a number of interesting events involving productive uses of reading by the students that did not fit our definition of an RLM episode. For example, at the beginning of the "Math Connections" course, students watched a video in class while recording ideas on index cards (a variation of the "cloning an author" reading strategy [Harste & Short, with Burke, 1988]); we decided that, although this instructional episode clearly involved a transactional reading strategy, it should not be counted as an RLM episode because it did not involve a written text. Another example of an episode that we decided not to "count" as an RLM episode was an instructional event in which the teacher in the "Math Connections" course held up a children's book and read a few pages from it to illustrate a possible math connection; although this episode undoubtedly involved a "rich" text, it was not read using a transactional strategy, which was an essential part of our definition of an RLM episode. On the contrary, the group was unanimous in deciding that translating into words a "fictitious Greek manuscript" representing various diagrams to derive area formulas without any verbal explanations (see "Area Diagrams"/G3 episode, described in the Appendix) should be considered an RLM episode. We agreed that a text without words could still be considered a rich text and the way students worked with this text was a good example of "transmediation" (Siegel, 1984), as it involved the transformation of the original text using a different symbol system (i.e., from diagrams to words). In sum, these discussions helped us further refine our definition of an RLM episode and clarify our interpretation of "reading rich math texts generatively" more generally - especially the awareness that this is only one of several possible ways to conceptualize the role of reading in mathematics instruction, a point we will return to in a later section.

At the conclusion of this process, we had identified a set of 18 RLM episodes which occurred in the context of the 5 RLM units - 5 in the geometry unit (codes: G1 - G5), 3 in the probability unit taught in the suburban class (P1 - P3), 1 in the logic unit (L1) and 1 in the probability unit taught in the urban class (p1), and 8 in the Math Connections course (C1 - C8). A brief description of each of these RLM episodes can be found in the Appendix along with the abbreviated titles and codes that will be used to refer to specific episodes in the remainder of this monograph.

For each of the RLM episodes thus identified, all the available data were put together and a narrative of the event prepared whenever one was not already available. These data were then examined in order to identify each instance when reading occurred and, for each of these cases, *what* the students read, *why* and *how* was recorded in a table form (for an example of these tables, see Tables 7.1 and 7.2 reported later in Chapter 7). Once again, this analysis was conducted by a researcher who had participated in the experience and the results reviewed by another researcher, with disagreements resolved by consensus after discussion.

Based on this preliminary analysis, all four researchers then reviewed the entire set of RLM episodes in order to select a small subset that would well represent this corpus; at this point in the analysis our goals was to create thick descriptions that would show what "reading rich math texts generatively" looked like in practice and allow us to generate some initial categories related to the three research questions noted earlier. The two episodes reported in the next chapters were chosen because we agreed that, taken together, they fully illustrated the potential of reading rich texts generatively, explicitly addressed the range of learning goals we were interested in supporting, involved variations of the four major transactional strategies we identified, and represented the first time these particular strategies were introduced to that group of students.

The complete set of data available for these two RLM episodes (including field notes from classroom observations and planning meetings) was examined further by all four researchers with the goal of generating categories related to the three research questions previously identified -- i.e., (1) What, why, and how did students read in these experiences?; (2) How did the classroom teacher orchestrate such reading experiences?; and, (3) What did the students gain from these experiences? - as well as further insights about "reading rich math texts generatively". The results of these independent analyses were discussed by the whole group, and revised, elaborated and consolidated as a result of this discussion. It is important to note that the thick descriptions reported in Chapter 4 were created only *after* this analysis was completed, in an effort to highlight the findings of our analysis and include all the data necessary to substantiate these findings (reported in later chapters of this monograph) while at the same time reducing the original complete narrative to a more readable length. These considerations also led us to include only a portion of the original "Fig Leaf 1" episode in the first vignette -- hence the characterization of that vignette by the short title "Math & War" to distinguish it from the entire RLM episode from which it was derived.

The other 16 RLM episodes were then reviewed in light of the categories generated from the in-depth analysis of the two selected RLM episodes in an effort to find additional support and/or discrepant cases (Erickson, 1986). In this stage of the analysis, we reviewed the entire set of RLM episodes searching for evidence of the occurrence/non-occurrence of the categories generated from the analysis of the two RLM episodes. For each RLM episode, one researcher who had participated in that experience examined the data and recorded not only whether or not the episode reflected an occurrence of that category, but also the reasons and evidence for this decision; the results of these analyses were then compiled in the summary tables, reported later in the monograph (see Tables 6.1, 7.3, 7.4, 8.1 and 8.2). To aid in the interpretation of these tables we recommend that readers skim through the Appendix and read at least a few of the descriptions of RLM episodes reported there, so as to get a sense of the variety of reading experiences that were generated.

4. What Does "Reading Rich Math Texts Generatively" Look Like? Two Prototypical Examples

The two classroom vignettes presented in this chapter are an attempt to provide a powerful illustration of what it means to read rich math texts generatively in practice. As mentioned in the previous chapter, they were selected because, taken together, they address the range of learning goals we are interested in supporting, illustrate the use of a wide variety of texts and variations of all the three transactional reading strategies ("say something", "cloning an author" and "sketch-to-stretch") discussed in the reading literature as well as other generative uses of reading mathematics, and represent the first time these strategies were introduced to that specific group of students. Furthermore, these two experiences were carried out in quite different instructional settings (an experimental course in an alternative urban high school and a "regular" eighth grade math course in a rural/suburban school, respectively). Finally, the two instructional episodes deal with quite different mathematical content as the first vignette focuses explicitly on learning about the nature of mathematics whereas the second addresses mathematical content typical of the traditional curriculum.

Though these vignettes are considerably longer than the instructional vignettes usually included in research reports, there are several reasons for this. First of all, we felt it was essential that students' interactions with the texts and with each other be presented in sufficient detail in order to appreciate how transactional reading strategies actually played out in these mathematics classrooms, how different students used them and felt about them, and what individual students gained from these experiences. Furthermore, we wanted to show how these reading strategies were used and orchestrated in relation to larger instructional activities, which had been constructed to achieve specific instructional goals; separating the reading strategies out from these activities would have made it more difficult to see how the reading strategies contributed to the mathematical thinking and learning taking place. It is also important to appreciate that, given the instructional goals they were intended to support, these activities often developed over more than one class period; in fact, each vignette describes an instructional episode that took about 2 hours of instruction. Finally, the detailed nature of the vignettes reflects their dual purpose, that is, to show what "reading rich math texts generatively" looks like in practice while at the same time serving as a data base that could enable the reader to understand and find empirical support for the more systematic analysis of this approach to reading mathematics which will be reported in later chapters.

4.1. Exploring the Connections between Mathematics and War through Reading, Talking, and Drawing ("Math & War" vignette)

This vignette illustrates two readings of the essay "Mathematics and War" (Davis & Hersh, 1981), both of which involved the use of the "say something" and "sketching" strategies by students working in pairs or groups of three. The vignette focuses especially on the reading experience of two students, Char and Jolea, and offers an especially rich example of the negotiation and generation of meanings through social interaction, as well as a spontaneous evaluation by Jolea of the effectiveness of the strategy. Later in the vignette, however, we will make reference to two other students, Van and Shellie, in order to point out some similarities and differences between the reading experiences of these two groups of readers.

The vignette took place in the context of the semester-long course on "Math Connections" taught in the urban alternative high school after the students had planned and carried out their first inquiry into the math connections involved in various types of racing, which culminated in a series of group presentations. Neither the students nor the teacher had been completely satisfied with these presentations. As the students themselves noted, the groups had not been very successful in integrating the findings of individual group members into a coherent presentation that could be understood by others. Working as a collaborative teaching team, the classroom teacher, mathematics education researchers (RM hereafter), and the reading education researcher (RR hereafter) therefore decided to introduce a variety of reading strategies to help students develop ways to solve this problem in future inquiries. Each of the strategies they chose, starting with the "say something" strategy, invited students to share their thinking with others and build connections between and among the ideas that were shared.

Their plan for introducing the first reading strategy was to have students select, and then read using "say something" one of two sections, "Mathematics in the Marketplace" and "Mathematics and War", of the essay "Under the Fig Leaf" from The Mathematical Experience (Davis & Hersh, 1981). This book explores the nature of mathematics, including its social, historical, and political dimensions. To give the students an idea of what the "say something" strategy was all about, the RR asked them first to read silently the short paragraph that introduced the two articles.

***TEXT:** A number of aspects of mathematics are not much talked about in contemporary histories of mathematics. We have in mind business and commerce, war, number mysticism, astrology, and religion. In some instances the basic information has not yet been assembled; in other instances, writers, hoping to assert for mathematics a noble parentage and a pure scientific existence, have turned away their eyes. Histories have been eager to put the case for science, but the Handmaiden of the Sciences has lived a far more raffish and interesting life than her historians allow.*

The areas just mentioned have provided and some still provide stages on which great mathematical ideas have played. There is much generative power underneath the fig leaf. (Davis & Hersh, 1981, p. 89)

The class took about two minutes to read the paragraph to themselves and was immediately ready to "say something." RM moved to the front of the room to record the students' responses on newsprint as the discussion slowly unfolded. "It reminds me of a Congressional Report--it's a bunch of bureaucratic mumble-jumble," said the first student. "I didn't understand it at all," added the next. "Too many big words," agreed a third. Gradually, the students' questions and comments shifted to the intended audience and purpose of the piece. "Who'd they write it to?"; "What's the point?"; "Is this the whole thing or is there more to it?" RM quickly recorded these responses on newsprint while RR asked if someone could say something about the content of what they had read. One girl responded with more questions, "What are they trying to do in here? Are they trying to make some kind of connections between science and math?" Others focused on the title. "Under what fig leaf?" they wanted to know. Finally, a boy who had been rather quiet up to this point, said, "I think I understand. It's suppose to be like what this class is about. Because it is talking about how people don't realize that math is related to other things." RR took this opportunity to stop and point out all the important things that happened in their "say something" conversation, in an attempt to help the students value the process they had just experienced. She especially wanted to emphasize the process of negotiation they had gone through to make sense of the material--how they had begun with frustration over the language of the piece but had gradually built off one another's questions until the connection between the author's message and their own mathematics class had become clear.

The next time the class met, the students quickly paired up and chose one of the articles mentioned above. The pairs then moved to comfortable spots in the room, and settled into the rhythm of reading and talking with their partners. Char and Jolea had selected "Mathematics and War" and, after some initial hesitation, they agreed to let RR participate in their "say something" conversation. (Note that RR's role in this reading experience was intended to be that of a partner, although at times she also implicitly modeled for the other group members some behaviors and roles that she hoped the students would eventually learn to do on their own.) The two students decided to proceed by reading paragraph by paragraph, just as in the whole class demonstration, and started reading the first paragraph silently, though they found it hard to concentrate with another student reading aloud across the room.

TEXT: Legend has it that Archimedes put his science at the service of warfare. He is reputed to have devised compound pulleys to launch galleys, to have invented a variety of catapults and military engines and most spectacularly to have focused the sun's rays on besieging ships by means of a paraboloidal mirror. All this for King Hieron of Syracuse who was the most brilliant scientist and mathematician of his age, but the achievements just listed, although they can be explained by the mathematical theories of mechanics and optics, do not appear to have involved mathematics at the basic level of application. (p. 90)

After another brief interruption, they paused to talk about the first paragraph.

Jolea: To me, I don't know much about the subject [war]. It is.. I don't know. It's kind of mixed up. I don't know. It's.. Maybe I'm just stupid. (Char is mumbling.)

RR: (responding to the last comment) No. Well, I guess I was surprised that it went back that far. That connection between war and mathematics. I mean all the way back to.. I mean that first sentence: "Legend has it that Archimedes put his science at the service of warfare." Even way back then. What were you saying?

Char: Well, the last of it said they did not appear to involve mathematics in the basic level of application.

RR: What do you think about that?

Char: Well, I don't know. Kind of like it happened but we didn't think of it.

RR: Yeah, I thought that was strange too. Because I thought that it was pretty clear that catapults and adjusting parabo[loidal] mirror to get the sun's rays and reflect that back. They seemed to involve mathematics. Maybe I'm.. just because I've been in this class I see that connection. But they seem to be saying that umm.. that it doesn't [involve mathematics]. So. What were you saying, Jolea, before?

Jolea: I was just.. I don't know I didn't understand while you were talking but now I understand. I guess a lot of the words I didn't understand.

Char: I couldn't pronounce them. (They both laugh about the difficult words, such as Archimedes.)

After a bit more discussion, RR suggested they try the next paragraph.

TEXT: What is the relationship between mathematics and war? In the beginning, the contribution was meager. A few mathematical scribes to take the census and to arrange for induction into the army. A few bookkeepers to keep track of ordnance and quartermaster. Perhaps a bit of surveying and a bit of navigation. In their capacity as astrologers, the principal contribution of the ancient mathematicians was probably to consult the stars and to tell the kings what the future held in store. In other words, military intelligence. (p. 90)

Jolea's immediate response to the paragraph was "Confusing." Char agreed and commented on how the paragraph jumped from saying the relationship between mathematics and war was small, to talking about consulting the stars and astrology; "it didn't feel like it fit," she concluded. Jolea focused on how mathematicians didn't seem to have a distinctive role in early times and that their roles were, instead, combined with other things; later she shared her view that astrologers were the "total scientist," which led Char to respond that she thought of them as "corny people that don't know what they're talking about." RR's suggestion that perhaps this image had been different in earlier times led Jolea to begin thinking about the difficulty of separating mathematicians and scientists and wondered if astrologers were mathematicians since they were considered the scientists of their day.

They read the next paragraph for several minutes before picking up their conversation.

TEXT: Modern warfare is considered by some authorities to have begun with Napoleon, and with Napoleon one begins to see an intensification of the mathematical involvement. The French Revolution found France supplied with a brilliant corps of mathematicians, perhaps the most brilliant in its history: Lagrange, Condorcet, Monge, Laplace, Legendre, Lazare Carnot. Condorect was a Minister of the Navy in 1792; Monge published a book on the manufacture of cannons. Under Napoleon, mathematicians continued to bloom. It is reported that Napoleon himself was fond of mathematics. Monge and Fourier accompanied Napoleon on his Italian and Egyptian campaigns, and if these men did not do anything directly mathematical during these army hitches (Monge supervised booty while Fourier wrote the Description of Egypt), one is left with the feeling that Napoleon thought that mathematicians were useful fellows to have around. (p. 90)

Jolea responded to RR's request to "say something" by "confessing" that she hadn't understood the paragraph and had therefore "cheated" and read it several more times. RR quickly refuted the notion that rereading was "cheating," and Jolea noted that rereading enabled her to understand the paragraph. Once again, the "say something" consisted of each reader offering her interpretation of the text and responding to one another's ideas. Char began with her view that the text was just saying that Napoleon was a great guy and had recognized the value of mathematics and that was good. RR connected back to the previous paragraphs and noticed a progressive increase in the use of mathematics during war, which was heightened during Napoleon's times, in part, because he had a personal interest in it. This observation prompted Jolea to ask for help in making sense of the last part of the final sentence in the text: ... *one is left with the feeling that Napoleon thought that mathematicians were useful fellows to have around.* Jolea wanted to know if that meant Napoleon felt mathematicians were useful to him personally, to which Char responded that she thought it meant useful for building knowledge since they didn't have colleges to attend. RR agreed and referred back to examples in the paragraph that gave specific examples of the kind of knowledge they might produce (e.g., a book on cannons). Jolea then focused the group's attention on the first part of the sentence that confused her (... *and if these men did not do anything directly mathematical during these army hitches*) and said she "took it as useful to him," an interpretation that RR seconded.

There was another two minute pause while the participants read the next paragraph.

TEXT: Arriving at World War II, one finds mathematical and scientific talent in widespread use in the Army, Navy, and Air Force, in government research laboratories, in war industries,

in governmental, social and business agencies. A brief list of the variety of things that mathematicians did would include aerodynamics, hydrodynamics, ballistics, development of radar and sonar, development of the atomic bomb, cryptography and intelligence, aerial photography, meteorology, operations research, development of computing machines, econometrics, rocketry, development of theories of feedback and control. Many professors of mathematics were directly involved in these things, as were many of their students. This writer was employed as a mathematician-physicist at NACA (later NASA), Langley Field, Virginia, with only a bachelor's degree to his credit, and many of his contemporaries at Langley Field subsequently occupied chairs of mathematics throughout the country. (p. 90-91)

Interestingly, Char commented that the text was getting easier for her to read, despite the use of such words as 'cryptography' and 'econometrics.'

Char: It was easier to read than the other.. three [paragraphs].

RR: It's getting easier, huh?

Char: Yeah, I don't know if it was just the words or if I connected to it more. I felt like when they were going through the different umm.. things -- aerodynamics, da da da -- I.. every time I read another word it [was] like, "Yeah, that's related." And I [thought] how it was related and I would go on.

Jolea: I saw it as a growth.. in math. They started to connect other things that they could do .. other things that math was involved in. It went into careers..

The group spent about three minutes reading the next paragraph.

TEXT: With the explosion of the atomic bomb over Japan and the subsequent development of more powerful bombs, atomic physicists who had hitherto lived ivory-tower academic existences experienced a sense of sin. This sense of sin spread simultaneously over the mathematical community. Individual mathematicians asked themselves in what way they, personally, had unleashed monsters on the world, and if they had, how they could reconcile it with whatever philosophic views of life they held. Mathematics, which had previously been conceived as a remote and Olympian doctrine, emerged suddenly as something capable of doing physical, social, and psychological damage. Some mathematicians began to compartmentalize their subject into a good part and a bad part. The good part: pure mathematics, the more abstract the better. The bad part: applied mathematics of all kinds. Some mathematicians and a rising generation of students left applications forever. Norman Wiener, who had been engaged in developing theories of prediction and feedback control, renounced government support of his work and devoted the remainder of his life to doing "good works" in biophysics and to propogandizing against the nonhuman use of human beings. (p. 91)

Char and Jolea started by sharing their confusion with this paragraph, but this time, Jolea explained that she used the extra time she had to reread the paragraph. She began to talk about the understanding she had gained from rereading, which created a conversational scaffold for Char, who jumped in with her thinking. Once again, the group was able to build off one another's talk to negotiate and construct meanings.

Char: I understood the first inch of it, but then I must have got lost. (RR asks "Inch?") Maybe the first five, six lines -- for the rest of it I like "Wow."

Jolea: I.. I got lost up here but when you walked away, I just said well I've got time now, I'll just do it over again. And I got it and I understood it. And.. I think it's something that we forget a lot of the time. So many times praising what the scientists come up with and learn the uses [of], that we forget about the bombs and stuff and that scientists.. Because of them and their knowledge that's what comes out of it sometimes. The bad, just the bombs and..

Char: Yeah, but we can't have the good without the bad.

Jolea: Right, exactly. But I don't think.. I think it's more focused on other things and we forget. Yeah, we talk about nuclear war a lot but we forget what it comes from. I mean it's just not there. And to think so many people have that knowledge -- it's scary. Some people could do so many horrible things with it.

The group talked more about the guilt the mathematicians felt but also discussed the fact that it might be more dangerous for one country to have nuclear arms than for both to have them, implying that there was more than one way to think about the development of weapons. The group went back to reading, and when Char walked away for a minute, Jolea offered a spontaneous evaluation of the "say something" strategy.

Jolea: I don't know if I should say this but.. I don't think I'd be able to read from a book [like this] without reading and then talking about it.

RR: Why do you think?

Jolea: Because I think I've tried reading some and I get so confused and..

RR: You get confused? What's helping about this?

Jolea: That you only read part .. You know, usually you have to read the whole thing, maybe you do it for homework and then the next day it's not as fresh and you forget the little things that.. You remember a couple of things that confused you. But now it's.. and you're doing it right away and something's right there.

RR: Could you see a way to adapt this while you are doing some homework stuff?

Jolea: Yeah, I think eventually you can. Yeah. That's another thing I was going to say, I think eventually if you do this enough to learn.. Just like anything if you do something with someone enough you can usually go solo.

When Char rejoined the group, they continued discussing the value of the strategy before deciding to move on to the next paragraph which explored the galvanizing effect of Sputnik on the relationship between mathematics and war. The group talked about how the U.S. had responded to the launching of Sputnik, which prompted Jolea to comment that "almost as if the paragraph before [on the moral consequences of mathematical research] was forgotten." Since they were running out of time, RR suggested they read the rest of the article, which described the protests, some of them violent, against the scientific institutions thought to have contributed to the Vietnam War. The last paragraph played an important role in this conversation and in the one two other students reading this article, Van and Shellie, had as well.

TEXT: One began to hear it said that World War I was the chemists' war, World War II was the physicists', World War III (may it never come) will be the mathematicians' war. With this, there entered into the general consciousness the full realization that mathematics is inevitably bound up in the general fabric of life, that mathematics is good or bad as people make it so, and that no activity of the human mind can be free from moral issues. (p. 92)

As the conversation unfolded, Char tried to make sense of Jolea's concern with the moral dimensions of mathematics and war and Jolea continued to work on sorting out the relationship between science and mathematics.

Jolea: Umm.. I was still stuck on seeing the guilt of the scientist.

Char: I never saw that.

RR: The guilt?

Char: Yeah..

Jolea: Yeah.. the guilty.. But it seemed to.. [the guilty feelings] went to the people who weren't involved in science. They were.. Once the guilt went away, so it seems, or maybe the people who did feel guilty just backed away and just said, "Forget it.
[I' m not going to do] this type of science." But ahh.. Just the..just started to condemn the ones that were doing it.

RR: Well, and that gets really complicated. Because on the one hand we don't want to get rid of science.. (Jolea agrees) and mathematics. And yet on the other hand that's a dilemma. How do you use it in responsible ways?

Jolea noted that this dilemma was still with them and described a discussion in another class on Reagan's "star wars" proposal; Char reflected on the question of environmental impact the class had raised and commented that it was hard to predict the moral consequences of something until you're working on it. When RR asked what they thought of the sentence that assigned each world war to a different scientific discipline, Char responded that it had surprised her because the text had built an argument that they had all been mathematicians' wars and now they were separating them, whereas for Jolea the overarching question continued to be the division between science and mathematics. At this point, RR gave Char and Jolea the directions for the sketching strategy, which they were to complete for homework. She asked them to make one sketch that showed what they had learned from the text.

At the next class meeting, students who had completed the sketching assignment were asked to explain their drawings to the rest of the class. Char and Jolea began that by explaining that they had been unable to create one sketch because they had "disagreed too much," that is, each of them had come away from the reading experience with a different theme they wanted to explore. There were similarities between their sketches in both form (both were divided into small boxes, some of which were further divided by a diagonal line, and used cartoon-like stick figures) and content (both represented the increasing use of mathematics for war and the moral questions it raised). When it was Char's turn to share, the teacher wanted her to pass her sketch around the class so everyone would have an idea of what she was talking about, but Char felt she needed it in her hands to explain it, so it remained in her hands as she spoke (see Figure 4.1).

Char: It started out with the catapult. ... When I saw the catapult I see.. strictly math as I guess I've always known, a lot of numbers and a lot of angles. And then having the scientists being ... really proud of themselves but also the society being really proud of ... a group of people being able to think that stuff up. And then it went on to like the atom bomb. And h~ v the majority of the scientists ... thought that was great, "Look what we can do now, look at the technology we have." But then there was a few of them that were kind of like, "Why am I doing this? All I'm doing is killing people." And society kind of started to ah.. realize what was going on. There was a few of them around that didn't like the idea but the majority of them thought that was good. You know, "We bombed China. We did this." Now with nuclear war there's still a lot of scientists that [say], "Wow look at the technology of fusing atoms and everything." But there's more scientists now that feel that's wrong to just do this ... research just to build more warheads to beat Russia. And in that case there's a lot more of society that feels it's wrong.. for the scientists to be doing this because we are only killing people. But there's still a few that are, "Yeah, nuclear war is needed." And that's what my drawing shows.

Char's sketch thus went beyond the text and showed three aspects of the connection between mathematics and war: the first column showed the actual product of the mathematicians' thinking at three different points in time (e.g., a catapult, a bomb, "star wars"); the second column showed the mathematicians' moral dilemma (between the intellectual challenge of doing the research and the effects on humankind); the third column showed society's response to the products of the mathematicians' research, both pro and con. At this point Char passed her sketch around. After discussing the teacher's questions about the comic strip motif, Jolea's sketch (see Figure 4.2) was sent around the class with a warning from her that it might look a lot like Char's but "the feeling behind it" was different. Upon hearing this, the teacher reworded her directions to the class, suggesting that the students look more closely at the sketch to see the differences.

Jolea: ... In the beginning I didn't look at it as really war. I looked at as people then separating people, professions like scientists and mathematics. Not being able to connect mathematics to anything, really -- they were very small, the connections, I think, very small. ... And it seemed to be very science involved -- science did not connect to math. And this is why in my second picture there was a dividing line and there was like a [black]board up here. This [the blackboard] means math and this [a lab table] means science and there's the dividing line [between them]. And then Napoleon came along and then he kind of brought those two together. From what I understood from the reading, he brought math and [science] more together but yet still ... they still didn't connect very well. ...

Teacher: So is that why the dividing line [in square #3] is missing? (They looked at the sketch again.)

Jolea: And then more doors started opening up so everyone was happy. You know, they were coming up with new things and everyone was happy. And they also started working on the atomic bomb but when the bomb came along.. The way I took [it] was after the atomic bomb scientists [felt guilty about this application of mathematics to war].. she said it wasn't so many scientists and you see my scale, there was to be a greater amount of scientists that knew what they had done.. not only what they had done but what they were teaching other people? And maybe they are not as ethical.

Jolea seemed a bit unsure of herself as she talked about this part of her sketch and began to talk more about the process of reading and drawing. As she talked, it became clear that her sketch reflected her question regarding the dividing line between mathematics and science as well as the moral dilemmas that arose from the application of mathematics to the practice of warfare.

Jolea: Then.. see, I split it up only because of the way I read it, I think. We read it paragraph by paragraph and discussed it. After each paragraph we discussed it and I think this is why I came up [with] these divisions. Because after it said that the scientists weren't thrilled with what they had done and then started to regret it, it went on to the next paragraph. It was like that was all forgotten. They didn't care what they did -- they were getting money, you know. They were coming up with these things like that's all they cared about. That's where the nuclear comes in. ... They just didn't care anymore but then society started to care and they started the protests. And that's were this picture comes in [square #8]--society is unhappy and scientists are happy.

After some discussion of the difference between doing mathematics and using mathematics, other students began asking questions about the sketches and the "say something" experience. Krista's question was generative for both Jolea and Char as it gave them a chance to reflect further on the experience.

Krista: What did you learn about yourself?

Jolea: That I can't separate math and science. That maybe there is no.. Maybe not about myself; maybe something that I made myself realize. I liked the article. It was really something to think about.

Several people then asked Char the same question.

Char: Well, I guess I knew all these. I knew about the catapult. I knew about the atom bomb. I know about science and I know about society. But I never put the three of them together like a story. "This because of this then this and then this happened." But I knew them as little separate stories -- I never like made one big thing out of it. So actually seeing it here -- kind of like, WOW why didn't I think of that!

It is interesting to contrast Char and Jolea's reading of this essay with that of Van and Shellie, another pair who chose to read the same article (see Siegel et al. [1996] for a more in-depth description of this reading event). Van and Shellie's approach to the "say something" strategy was slightly different, but equally helpful in supporting their efforts to make sense of the text. What was striking was the way Van and Shellie defined particular roles for themselves, Van reading aloud and Shellie interrupting to ask questions about what the text meant. Though at first glance this may make it seem as if Van was active and Shellie passive, that was not the case, as Shellie often responded to her own question with a possible interpretation or a connection that gave the ideas represented in the text more meaning. For example, when Shellie didn't understand what the word 'catapult' meant, Van's explanation made Shellie think about an episode from the cartoon "The Smurfs" in which Smurfs had used a catapult. When they got to the section of the text that discussed the protests against the role of mathematicians in the Vietnam War, Van made a connection to the environmental organization Greenpeace and remembered an episode from a television show in which an abortion clinic had been bombed. And, as they thought about the last paragraph of the text, Shellie tried to understand the idea that World War I had been the chemists' war by relating it to something she had learned about mustard gas in another class. Thus, by bringing their own background knowledge to bear on the text, Van and Shellie were able to develop a personal interpretation. Their discussion of the last paragraph offers an especially good illustration of how these two students approached their "say something" and in particular of the way they built connections between the text and their own lives.

TEXT: One began to hear it said that World War I was the chemists' war, World War II was the physicists', World War III (may it never come) will be the mathematicians' war. With this, there entered into the general consciousness the full realization that mathematics is inevitably bound up in the general fabric of life, that mathematics is good or bad as people make it so, and that no activity of the human mind can be free from moral issues.

Van: So, what do you want to discuss? ... Not World War III because there is nothing *(laughs)*. I don't want to discuss World War III.

Shellie: I want to know why they said World War I was the chemists' war, why they said World War II was the physics war.

Van: I don't know! Okay, okay, let's go back.. World War I..

Shellie: Chemicals were used in World War I? Ohh.. The gases.. remember, Charles said the yellow gases? *(she refers to information about mustard gases gathered in another class)* It was because.. remember, because chemicals were used in World War I and it took a mathematician to mix the chemicals together?

In the meantime, Van is looking back through the article trying to find any reference to World War I:

Van: I don't remember having World War I in here... That's World War II. ... No. I'm trying to find it in the back.

Shelley: Oh, they never said anything about it.

Van: Then, how are we supposed to figure out what chemist was involved besides the mustard. *(they both laugh loud).*

Shellie: Okay, let's say World War II then. How was it the physicists' war?

Van: They built the bomb.

Shellie: All right Vanny! Then why will *(reading from the text)* "World War III (may it never come)" be mathematics?

Van: 'cause I think that it would take a lot of math and stuff to build something stronger than the atomic bomb.

Shellie: *(Jokingly)* Please we've got all the weapons... Man, we would like dog Russia.. We would dog everybody. *(they laugh, then more seriously Shellie asks)* See, why would we need mathematicians? We've got all the s... we need to blow up the world!

Van: No, because let's say.. Let me try to put this into cars. Okay. I'm going to do this for you. All right. World War I, your first car.. is a Chevette. *(at first Shellie makes fun of the analogy, saying that World War I should be a go-cart; Van continues).* No, listen. World War I: Joe's brown car, the Buick Regal when he first got it. Now, he bought it like that -- World War I, people did not have many ideas -- it came like that. Now World War II -- Joe got it black!

Shellie: He painted it and... *(both girls add details)*

Van: He hooked up a little bit of the engine. His seats were still brown, remember? So it's ... got the tint, but he don't got the phone or nothing yet, okay? So World War II he hooked up a little. World War II they hooked it up a little *(meaning weapons, etc.).*

Shellie: Okay.

Van: World War III -- after the regal.

Shellie: The shifter, the phone, the ... *(a few more car improvements are mentioned)*

Van: But World War III, okay, after it's all hooked up; he's cruising, he's sporting it hard. Now he's thinking of more ideas ... Like World War III, these people.. Now, he wants to beat his Regal, okay, he wants something that is better than his Regal. They want something that is better than the atomic bomb. Because if Russia builds an atomic bomb that's better than us, then we have to think of something better than them -- to keep up with the competition, okay?

The two students continue the conversation on these lines for a while, and finally Shellie concludes:

Shellie: That was a good example though, Van. That was really good.

Van: What, Joe's car? I knew you..

Shellie: I understand it. ... We should tell someone about that.

Van: Yeah, we compared our math to Joe's car. I knew he was good for something in school.

When Van and Shellie created their sketch, they chose to represent the analogy that Van had created between the increasing role of mathematics in warfare and the improvements their friend had made to his car over time (see Figure 4.3). Sketching allowed them to develop this analogy even further and generate the idea that, just as Joe could continue making improvements to his car, mathematics continues to develop as well. As they later noted, "It's almost like a question, too, because there's like more things that he can do with his car and there's more ways that math can be used".

Notice how drawing what they learned from the article gave both pairs of students an opportunity to think further about the text from a different perspective and, in this way, complement their previous "say something" experience. Whereas "say something" helped the students turn what might have been a frustrating and superficial reading of the essay into a collaborative meaning-making experience, "sketching" invited further reflection and elaboration of their interpretations. Together, these strategies enabled the students to demonstrate to themselves that there is more to reading than "mining" the text and more to learning than thinking in isolation.

4.2. Investigating Technical Concepts and "Big Ideas" about Geometry through Multiple Readings of an Essay ("Egg Man" vignette)

This vignette shows how a teacher engaged an entire class in multiple readings of an essay that examines several important geometric concepts in the context of a contemporary real-life problem. Each reading of the essay was accomplished using a reading strategy adapted to fit the teacher's instructional purpose. As a result, this vignette illustrates a variation of the "cloning an author" strategy (hereafter the "using cards" strategy) as well as an original strategy we called "enacting the text," which the teacher spontaneously introduced in a previous class to help make some of the mathematical procedures described in the text concrete for the students.

The instructional episode reported here took place in the context of the geometry unit taught in the suburban/rural middle school. Earlier in the unit, the class had used the "say something" strategy to read a text describing the kind of geometry that so-called "primitive" people might have used. By the fourth day of the unit, the teacher felt the class was ready to investigate some important geometric concepts in a contemporary situation. She thought that reading about this real-life event would help the students realize how geometry is used in everyday life even today, and at the same time raise the students' awareness and interest in the study of geometric figures -- a topic that she was planning to address explicitly later in the unit.

Over Thanksgiving vacation, the teacher therefore assigned the reading of "Adventures of an Egg Man," a 20-page essay included in the book *Archimedes' Revenge* (Hoffman, 1988) and dealing with the construction of a 3-story high "egg-monument." This "giant egg" was commissioned in 1974 by a Canadian town and supported by a grant from the Royal Canadian Mounted Police. As the author shows, this project proved to be much more challenging than anyone had initially expected, but was finally accomplished by a computer science professor named Resch who spent more than 18 months on the task. The essay very effectively portrays the struggles and creativity it took to solve the problem, as it explains the various approaches Resch tried and eventually abandoned (such as trying to come up with a method of construction based on the characteristic properties of an "ideal egg"--a shape that no mathematician had studied before!), before settling on constructing his "egg-surface" as a semi-regular tridimensional tessellation made of over 2,000 equilateral triangles and 500 equilateral 3-pointed stars. Throughout the essay, the author also presents a wealth of information about properties of various geometric figures and their construction, as well as other relevant mathematical facts and ideas (by means of non-technical explanations often involving diagrams), so that the reader can get a better sense of the mathematics involved in the solution of this real-life problem. In addition, some information about Resch's life and career (including other interesting examples of applied mathematics projects he engaged in) is presented, thus offering the reader some insights into him as a person and a mathematician.

To help the students make sense of such a long and difficult reading on their own, the teacher had thought of employing a variation of "cloning an author," which she introduced by explicitly connecting this use of 3x5 index cards with the "say something" strategy they had experienced in previous classes.

> Teacher: What we have done [so far] is to look at how people used geometry in primitive times, in the past. What I'd like to do [now] is give you a reading that shows you how geometry is used more currently. [...] Since you cannot work with your partner to do the reading over the vacation [...] I'm going to give you index cards. What I'd like you to do is write down some of your comments you would have made to your partner, had your partner been with you, on the index cards. [...] Just write one question ... one thing on each index card.

Despite some initial complaints about the length of the reading and about having such a long homework assignment over a vacation, all but four students completed the assignment. With one exception, the students used at least four of the 10 index cards the teacher had given each of them. What they recorded on these cards varied considerably in content, as illustrated by the following sample:

Student 1:

- What does an egg have to do with math?
- How are you supposed to build an egg with material that doesn't bend?

- Why does an egg symbolize peace?
- If mathematicians are so great why don't they find a formula to build an egg?

Student 2:

- How does this talk of chickens and a three story egg tie in with math?
- Did you feel that the way they made the ellipses and circles in the diagrams were quite unique?
- Is there a difference between an ellipse and an oval?
- If Resch's "changing of paper into three dimensional shapes is not origami," then what is it?
- I think it is interesting that sometimes, no matter how hard a person applies pressure, he will not be able to break an egg. Don't you feel that it's interesting also?
- I found that this reading was very confusing and hard to understand. Didn't you?
- At the end of the packet, it said that people wanted to blow up the egg. Why would they want to do that?
- A few paragraphs talks about how an egg is formed inside a chicken. What does this have to do with math?
- Why did everybody laugh at the idea of the egg?
- Why would he be interested in changing paper into three-dimensional form for two whole decades?

Student 3:

- Are the angles divided up so they always equal 360°?
- Are all the shapes equal even though their angles are different?
- Why does the "Banded Egg" need tiles?
- Are 2 identical triangles equal to a square if they fit in it? *(picture of 2 triangles, and then a square divided in half by a diagonal and forming the same triangles)*
- Are their angles the same? *(picture of two sets of equal triangles)*
- Even though an egg shape doesn't have an angle could you find out the degrees of its top? *(picture of the egg construction, picture of a square with an angle marked, and of an egg with the top marked as an angle)*
- How do you find the area of a circle or sphere? (p.83)
- How can the shape of an egg contribute to its strength? (pg. 84)
- Why did they do this? *(picture of two pins and a string attached to them forming an ellipse)*
- The story was hard to understand. Some of the words were hard but it wasn't that interesting.

Though the great majority of the students chose to write their comments in the form of questions (96 out of 118 cards). Most cards were used to record specific points from the essay that the students had not understood (e.g., "How can [3-pointed] stars be hexagons?"; "I don't understand the densest known sphere packing"; "Why does an egg symbolize peace?") or to raise questions about more general issues discussed in the essay (e.g., "What does the tiling requirement have to do with the shape of the egg?"; "Why did he have to look at so many eggs to try and find a perfect one?"). Some students, however, also recorded their impressions of the reading (e.g., "I like the way they constructed the egg like shape with thread, thumbtacks and a pencil"; "I found that the reading was very confusing and hard to understand. Didn't you?") and, in just two cases, what they learned from it (e.g., "Kissing spheres. I learned a little about what they are"; "I didn't realize the construction of a circle went back that far"). In a few cases, the comments went "beyond" the reading as the student challenged what was reported in the text (e.g., "Why not making something easier?"; "If mathematicians are so great, why don't they find a formula to build an egg?"; "Shouldn't an egg be bigger if it weighs 3000 pounds?") or, even more interestingly, raised some new questions (e.g., "What is the purpose of an Ellipse?"; "Even though an egg shape doesn't have an angle, could you find the degrees of its top?"). Interestingly, eight out of the 21 students also raised the question of "What does an egg have to do with math?"-- despite all the mathematical facts and explanations reported in the essay!

Though the first reading of the article occurred at home, about two class periods of 40 minutes each were devoted to follow-up activities related to the reading of the essay. First of all, the teacher asked for some volunteers to briefly state what the story was about and what they thought about it--to remind everyone, and especially the few students who had not done the reading at home, of the content of the story. A few students immediately volunteered, though most of them, rather than summarizing the content, tended to relate their impressions of the reading--mostly pointing out that there was a lot of information that did not seem relevant or related to mathematics.

Within this initial conversation, the teacher also invited the students to comment on how they did the reading and how they used the index cards. This elicited some interesting reactions and discussions. In particular, in response to a student's observation that there were "a lot of big words," the teacher tried to help the students appreciate that it is not always necessary to understand every single word in a reading . As she asked students to share what they did when they found words they did not know, several students agreed that they did not need to look them up in the dictionary, since in most cases they could make sense of the reading even if they skipped them, or could figure them out by looking at the surrounding words.

A student's question about what they were supposed to do with the cards they had written provided a natural opening for the teacher to introduce the small group activity she had planned so as to engage the students more actively in a discussion of the reading. The following instructions were written on the board (since in the past few lessons the students had seemed to have a difficult time paying attention to instructions once they were in their small groups), as well as briefly explained by the teacher:

Read index cards.
Arrange similar cards together.
Discuss what are the most interesting and important cards; choose 4 or 5 to discuss in your group.
Report to the class.

This group activity developed for about 15 minutes, and during this time most of the students seemed quite engaged and interested, though few groups got beyond the stage of classifying the cards and none managed to discuss the cards selected as most interesting (mostly because of lack of time). There was also almost no time left in this lesson to share the results of this group work, so the teacher collected all the index cards, asking each group to mark those they had selected as most important.

The lesson concluded with the distribution of an assignment sheet asking the students to "Look at the reading on Archimedes Revenge [the essay they had just read] and your textbook, list the names of as many shapes as possible"--in preparation for a future class activity where the teacher had planned to study systematically a few geometric figures chosen by the class from a compilation of the individual lists created in this homework assignment.

As the teacher read the index cards she had collected, she was pleasantly surprised by the variety and quality of the students' comments and questions--which told her that most students had seriously tried to understand the reading and got a lot from it, in contrast with the negative reactions she had gathered from the previous day's conversation in class. Thus, she thought that everybody would benefit from further follow-up activities based on the cards. Since she had concerns about the small groups' ability to do these activities on their own, she decided to try a different approach. First of all, she compiled a list of all the cards the groups had selected as "most interesting" (for the groups who had not done the selection, she herself chose what she thought were the five most interesting cards) and formulated the following homework assignment:

*Below are listed some questions from Archimedes Revenge, choose ONE question and find an answer to respond to the best of your ability to your classmate. (questions with a * before them are questions group members felt were the most interesting)*

** How can the shape of an egg contribute to its strength? (A. K.)*
** Why won't an egg break when squeezed on the ends? (J. D.)*
** Even though an egg shape doesn't have an angle could you find out the degrees of its top? (A. K.)*
** Are 2 identical triangles equal to a square if they fit in it? (A. K.)*
** Won't the circle not come out right if you tilt the pencil the wrong way? (p. 90) (A. K.)*
How can stars be hexagons? (K. B.)
What does the construction of a circle have to do with the myth about the egg? (M. S.)
What does the tiling requirement have to do with the shape of the egg? (M. S.)
What uses is there for a shape similar to an egg? (D. M.)
What is the purpose of ellipses? (D. M.)
How is the making of a real egg going to help him? (D. D.)
** How does 5 hexagons equal 7 circles? (C. G.)*
** Why didn't anybody use eggs in Geometry? (C. G.)*
** How do you construct an egg out of equilateral triangles and three pointed stars? (M. C.)*
Isn't an ellipse just like an oval? (R.. V.)
How does this talk of chickens and a three story egg tie in with Math? (T. D.)
Were the way they made the ellipses and circles in the diagrams quite unique? (T. D.)

This assignment sheet was handed out at the beginning of the next class, with the following explanation:

Teacher: I had a chance to go through all your cards, and can I just say at this moment that I was really impressed by some of your questions. I thought that they were very good. I really...I am truly amazed by some of the things that you're writing down. So I just wanted to let you know that, OK? [...] Now part of your homework assignment will be to pick one of these questions and try, to the best of your ability ... to give a good answer to one of those. Now you might have to look up some information. You might already know some information. Just try. Put down as much as you can to help answer one of these questions.

In order to provide a model for this activity, the teacher had planned to address two of these questions in class: "Won't the circle not come out right if you tilt the pencil the wrong way?" and "What does the construction of a circle have to do with the myth about the egg?" These particular questions were selected with the goal of helping the students appreciate some key points of the essay that were especially relevant to the geometry unit the class was engaging in (i.e., the characteristic properties of geometric figures and their relationship with methods of construction) as well as learning some properties of circles and ellipses included in the curriculum the teacher was expected to cover.

In preparation for this activity and while she set up all the material needed for this lesson and handed out the homework assignment., the teacher first asked the students to locate and re-read on their own the following section of the essay where the construction of a circle, an ellipse and an oval/egg are discussed:

TEXT: Resch soon found, however, that there was no formula in the literature for an ideal chicken egg. For many shapes that have a name, the literature contains not only an algebraic formula, but also a method of construction. Take the circle. It is simply the set of points in a plane that are equidistant from a given point in that plane. To construct a circle, tie one end of a length of string around a pencil and anchor the other end with a thumbtack to a piece of paper. With the string pulled taut and the pencil point help against the paper, rotate the pencil around the thumbtack; the result is a circle. (figure of a pencil drawing the circle) At some point, a twisted wag even turned this simple construction process into a sick joke, which I learned from the mathematician Martin Gardner. "Mommy, mommy, why do I always go around in circle?" "Shut up, kid, or I'll nail your other foot to the floor." It is an easy step from a circle to a sphere -- imagine the kid's foot (or the string's end) nailed to a point in three dimensional space, swing the kid's rigid body (or the pencil at the end of the taut string) every which way and observe the kid's head (or the pencil point) traces out. Alternatively, you can think of a sphere as the shape swept out by a pirouetting circle.
A chicken egg, of course, is closer to en ellipsoid -- the shape swept out by a pirouetting ellipse -- than to a sphere. Even the most demented mathematician wouldn't be able to generate an ellipse by twirling a child but could do so easily with the aid of a pencil and a loose string anchored by thumbtacks at both ends. (picture of a pencil drawing an ellipse) (pp.90-91)

The students did as asked, though they did not seem particularly engaged in the reading (not surprisingly, since so much was going on in the room at the same time and no explicit purpose for doing this reading had actually been given!).

The teacher then engaged the class in a concrete interpretation and testing of the first method of construction described in the excerpt, by asking the students to help her actually draw a circle following the procedure explained in the reading. As they did so, she initiated a discussion about the relationship of this method to known properties of circles:

Teacher: Well, they show you how to make a circle here, and what do they say about circles on that page? What's listed in that very first paragraph? What does it say? Can somebody just kind of summarize it for me? (Dave starts to explain.) Dave?

Dave: All the equal distances around one point will make a circle.

Teacher: Okay, so if you wanted to construct your own circle. You picked a center on the board here, and if you got a tight string... I'm going to need some help. Jessica, can you hold this for me? (The teacher has gotten a portable bulletin board and has set in on Jessica's desk. She is pointing to the board and showing the students exactly what she is doing to construct this circle.) I picked the center here, all right, I got the string...

The teacher tied a pencil to the string and then started tracing with it on the bulletin board, while the class watched with great interest. There were comments like, "Neato!" "Cool." "Ohh." "Interesting." "It's just like a compass."

Teacher: It's just like a compass. What happened here?

Class: A circle!

Teacher: Pretty good looking circle too, wouldn't you say Rick? (He agrees, as does much of the rest of the class.) What could I say is characteristic about circles? What kind of property? How are circles special, that allow me to do this?

Though several students started to answer this question, their contributions soon revealed that they had not yet made a clear connection between this construction of the circle and the defining property that "all points of a circle are at the same distance from the center":

Kathy: They don't have any angles or ..

Jessica: It's perfectly round, you don't have to like change. You can do it the same length away.

Teacher: What do you mean, "same length away"?

To help the students come to a clearer understanding that it is really the "equidistance" property that makes this construction possible, the teacher then suggested that they verify with a ruler that the distance to the center of each point of the circle they just drew was the same. The students seemed quite interested and engaged in this demonstration. The teacher also took the opportunity to review some vocabulary about circles during this discussion.

The teacher then explicitly revisited Kim's question ("Won't the circle not come out right if you tilt the pencil the wrong way?") by asking the student herself to demonstrate what she meant. Using the pencil the teacher had tied to the pin on the bulletin board, Kim began to trace a new "circle", but tilting the pencil differently as she moved around, while commenting "Like, if you tilted it like this it would come out all weird [pointing out to the rest of the class how the beginning and end of her figure did not even meet]. Like this."

The teacher tried to help the students see the significance of this result and its connection with the previous discussion:

Teacher: OK, notice. What did tilting this pencil do? What happened to the string? Did you notice when she was.. (One student says that it made an ellipse.)

Ron: It moved up on the pen.

Teacher: It moved up on the pen, which changed this distance, didn't it?

Ron: Yeah.

Teacher: It changed it--it almost made it shorter so that you didn't have a perfect circle, so that they weren't always an equal distance.

The teacher now moved to reproducing the construction of the ellipse--a figure the students had not studied before--as suggested in the reading (which, in this case, provided a picture with only little verbal explanation to illustrate this construction). This time, a student was asked to perform the construction on the bulletin board and he succeeded, despite some trouble at the beginning with moving the pencil around without upsetting the string. Then the teacher invited the class to come up with a characterizing property for this ellipse--as they had done before with the circle, though this time the information was not in the reading nor was it known to these students. The students had considerable difficulty addressing this question until the teacher suggested that they once again measure the distance of the points of the ellipse from the two pins. Several students were asked to measure these distances for a point of their choice, and these results were recorded by the teacher on the board in the form of a table. With some prompting on the teacher's part, the class then noticed that the sum of these distances was always the same number. The teacher concluded this activity by articulating the characterizing property of an ellipse.

At this point the teacher moved the focus of the discussion to the key issue of the relationship between the properties of a figure and its method of construction; she also helped the students see how looking at the construction of familiar figures can help with the construction of new ones, and in this way implicitly addressed the second student's question ("What has the drawing of the circle have to do with the myth of the egg?"). It is worth noting that the discussion that followed also addressed, albeit indirectly, some other questions various students had written on their cards--especially the ones challenging what the essay had to do with mathematics and this course in particular:

Teacher: All right, now, so what Resch said was that he thought this was interesting. This is an interesting property of ellipses, that the sum of those distances are always the same. And he thought that a special property of circles was that those points are always equal distance from that center point. Why did he mention this when he was mentioning the egg? What did he go on to say in those paragraphs? Look at p. 92. Read!

The students took some time to look back at the text.

Teacher: All right, what was he saying? [...] They said that there's things that we know about circles. There's properties that have already been studied and explored that we know about circles and ellipses. What did he say about eggs?

Dave: They don't have a similar property.

Teacher: They don't have a similar property and what else?

Dave: They don't have any properties.

Teacher: They really haven't been studied before. He found this very frustrating probably. If you wanted to build something and there was nothing to help you, wouldn't you find that a little frustrating? Don't you think that he'd have to do some work to find out some properties? And that's what he did. What did he go ahead and do? How did he try to find some properties about eggs? What did he do? (One student says, "Experiments!")

Ron: He drew it first.

Teacher: (Pointing to Ron) He tried drawing it but what did he do before he tried to draw it?

Ron: He looked up the different shapes.

Teacher: Where did he look them up?

?: [He found different eggs.]

Teacher: He got some different eggs, didn't he, together to just try and look and see visually what he could see. Just as if I brought in a whole bunch of circles and looked at them all first, maybe saw what was similar about them.

The conversation continued in this vein for a while, as the teacher tried to help the students understand the significance of Resch's story as an example of how mathematicians go about "discovering" the properties of a shape and using them for some purpose.

It is interesting to note that, despite the modeling that went on in class and the students' genuine interest in some of the questions and issues raised by the classmates and reported in the homework sheet, most students still had difficulty as they tried to address some of these questions on their own for homework. Some of their answers were quite superficial, although there were also several clever and insightful answers, as illustrated by the following examples:

Example 1

Question: What uses is there for a shape similar to the egg?
Answer: A similar shape is an ellipse. It is the shape in which the planets move, to describe stuff.

Example 2

Question: Why didn't anybody use eggs in Geometry?
Answer: I think maybe no one used eggs in Geometry or found a formula for it because maybe noone had really thought it was considered a shape. Maybe they just thought of it as a egg we eat. Or something in which a baby chick is hatched from.

5. A Discussion of the Previous Prototypical Examples from a Transactional Reading Theory Perspective

We believe that the vignettes presented in the previous chapter are powerful illustrations of new and varied ways that reading can be used to engage students in meaningful mathematics. In this chapter we will now examine these instructional episodes in light of the literature on reading theory and instruction so as to further explicate what it means to read "rich" mathematical texts generatively. At the same time, we want to point out some critical dimensions of the instructional practice of this approach to reading mathematics that emerged from our collaborative action research. In this way, our discussion of these vignettes will show how theory informs and explains practice and how practice generates new insights that can inform theory. This will involve three things: (1) a discussion of several key aspects of Rosenblatt's transactional theory of reading to show what these vignettes tell us about what this theory means in relation to reading in the context of mathematics instruction; (2) a consideration of those dimensions of instructional practice (adaptations and combinations of reading strategies, the use of sharing sessions) we came to regard as critical to the learning potential of generative reading in mathematics classrooms; and (3) a discussion of the challenge of working with "difficult" texts, an issue that raises important questions about students' perspectives on reading in mathematics classes.

5.1. Transactional Reading Theory and its Relation to Reading Mathematics in Practice

Though at first Rosenblatt's transactional theory of reading, described earlier in Chapter 2, may have seemed a strange choice for understanding reading in mathematics classrooms, the two vignettes presented earlier provide striking examples of Rosenblatt's theory in practice and suggest that a transactional perspective may indeed have something to offer mathematics educators. Three aspects of Rosenblatt's theory (the concept of transaction; the process of generating and revising meanings; variation in interpretation) will provide the focus of this discussion.

Rosenblatt defined the reading transaction as a unique event--a coming together of particular readers and particular texts in particular situations such that each element shapes and is shaped by the total situation (Rosenblatt, 1978). From this perspective, each of the reading episodes presented earlier was a unique, dynamic event so that to change any one element (the readers, the text[s], the social relations among the participants, the purposes) would be to create a new event. In the "Math & War" vignette, the situation focused on trying out some new strategies ("say something" and "sketching") to see if they could help students share information with one another in future inquiries; gaining a particular, predetermined interpretation of the text and displaying it for the teacher's evaluation was not the focus of this episode. At the same time, students were not just given any text to read but were asked to choose from a set that would help them develop their understanding of the course topic--math connections. Together, the "experimental" spirit of the situation, the invitation to share tentative meanings with peers as they talked their way through a text, the particular text selected, and the link to ongoing course discussions about math connections and ways of learning created a unique event. The uniqueness of each reading transaction becomes quite evident when we compare Char, Jolea, and RR's reading event with that of Van and Shellie's. Though the same text was read, each group did so in different ways, drew on different knowledge and experience, and thus produced different interpretations.

Char, Jolea, and RR's "say something" conversation often began with an assessment of their experience actually reading the text (such as "I don't know much about the subject," "Confusing," or "It was easier to read than the other three [paragraphs]") and continued with a discussion of what they thought the text meant (as when they tried to figure out if the statement *Napoleon thought mathematicians were useful fellows to have around* meant useful in general or useful to Napoleon in particular, or when Jolea tried to sort out the difference between the scientists and mathematicians, or when they examined the idea that there were moral consequences for producing knowledge). Often, their discussions took them beyond the text to their own experiences or knowledge (as when Char talked about how it was more dangerous when one government had the bomb than two, and Jolea made a connection between this conversation and one in another class on Reagan's "Star Wars" proposal).

For Van and Shellie the experience became a question and answer session, though, as noted earlier, Shellie often jumped in to respond to her own questions, thus rejecting the passive role she implicitly assigned herself! But their conversation was similar to Char, Jolea, and RR's in the extent to which they drew on their own knowledge to understand the text, knowledge that often came from both popular culture (e.g., television shows), other classes, and personal experience. One of the most interesting connections they made between the text and their personal experience was Van's analogy relating the evolution of their friend's car to the growing link between mathematics and war throughout western history. Going beyond the text to make this connection was essential to their understanding of the text and allowed them to gain new insights when they drew their sketch (i.e., the idea that math is like a question in the sense that there are more things their friend can do with his car and there are more ways that mathematics can be used). We can thus begin to see how the overall situation in which the reading occurred shaped the way these particular readers oriented themselves to the text and how, at the same time, the text provided cues that readers used to produce tentative interpretations and connections that could be negotiated with other readers.

The multiple readings of "Adventures of an Egg Man" described in the second vignette also illustrate the concept of a transaction among reader, text, and context in which each element shapes and is shaped by the others. The first transaction involved each individual student reading the text at home and recording comments or questions they would have shared with a partner during a "say something" experience. In this case, students approached the text with some prior understanding of the history of geometry and with the idea that they would learn something about contemporary uses of geometry from this text. Unlike the "Math & War" vignette, the focus here was primarily on exploring the ideas represented in the text rather than trying out a new reading strategy. This did not mean that students were expected to learn "everything" they encountered in the text, but, instead, to interact with whatever ideas they found intriguing while reading; later, they would have opportunities to discuss those ideas with their peers in class, and eventually try to answer their own questions. The range of questions and comments students recorded on cards suggests that they did treat this reading as an occasion for making personal meanings and not as an attempt to reproduce the text or arrive at the authoritative (read teacher's) meaning of the text. Still, the fact that 8 of the 21 students raised the question of what an egg had to do with math shows that they were definitely reading in light of a particular setting (their mathematics class). Being asked to read a text that seemed to contain extraneous information, that is, information unrelated to learning geometry, brought their histories as mathematics students into conflict with the overall situation in their mathematics class, a point we will return to later in this section.

A very different reading of the text was assigned for homework after students had shared their cards with one another and selected those they regarded as most "interesting and important." This reading transaction involved a search of the essay and their textbook for the names of as many shapes as possible. It is important to see how different this reading event was from the more open-ended reading they had just completed because of the difference in purpose. This time the reading experience was defined as a search for specific information rather than a meaning-making experience, a shift which shows quite clearly how the reading of the same text by the same reader can become a completely different event.

During the next class period, the students had yet another opportunity to read the essay on the egg man, but this time the focus was much more on developing shared understandings of the parts of the text that were related to the mathematical concepts the teacher wanted to pursue. Notice, however, that this did not become a transmission-oriented reading event in which the teacher delivered the content of the text to the students either through a directed reading or an oral summary of the points. Instead, she used several of the students' own questions to revisit the text and, for the first question, had them actually "act out" what the text described. This kind of active reading brought the text to life, literally, for the students and enabled the class as a whole to experience some concepts central to understanding what geometry is all about. The fact that only certain sections of the text were read in this enacting mode illustrates further how reading the same text can become a new event when read for a different purpose.

Another key aspect of Rosenblatt's transactional theory of reading is the idea that readers generate and revise meanings throughout the reading event. Interpretations do not spring ready-made from the text, even when each word is read accurately, but grow and change as readers propose tentative meanings and reconsider them in light of further reading, thinking, and dialogue. This point is illustrated in each vignette. The "Math & War" vignette shows how readers put forward tentative meanings and built off one another's thinking to arrive at an interpretation that made sense to them, whereas the "Egg Man" vignette shows how this same process of proposing and refining meanings took place across multiple readings of the same text, first with the goal of generating personal interpretations and later with the goal of constructing shared understandings about the connection between the properties of a shape (e.g., circle or ellipse) and the construction of that shape.

The whole class "say something" that introduced the experience provides a clear example of the way in which readers generated and revised their understanding of the introductory paragraph of the "Underneath the Fig Leaf" essay. Though their initial responses focused on the language of the text and the purpose for which it was written, their willingness to remain engaged in the process and propose tentative ideas and questions, even if they expressed frustration, enabled them to reflect on the text from different perspectives and eventually make a connection between the ideas represented in the text and their own experiences in the Math Connections course. This pattern was also evident in Char, Jolea, and RR's "say something" experience. Readers used the opportunity to stop and "say something" to express their confusions and surprises, summarize the text in their own words, interpret particular sentences and arguments, and make connections to their prior knowledge and experience as well as to earlier segments of the text. These comments were not a series of monologues, but built off one another, thus allowing each reader to construct, elaborate, or rethink her own hypothesis by considering the ideas of others. Char's reading of the article clearly shows how meanings grow and change through dialogue and reflection. If we were to look only at Char's sketch, which examined the conflicting moral responses that mathematicians and the public had to the development of modern weapons, we might conclude that Char had brought her own strong moral position to this text. Yet, throughout the reading event, it was Jolea who continued to raise this issue. Even toward the end of the article when Jolea commented on how she was stuck on seeing the guilt of the scientists, Char said she never saw it. Her sketch thus represents a rethinking of her initial interpretation and demonstrates how meanings are co-constructed through such dialogues.

The idea that meaning grows and changes through dialogue and reflection is also evident in the "Egg Man" vignette, though in this case the growth in understanding was achieved through multiple readings of the same text, each of which encouraged the students to take a different perspective. The first reading encouraged students to think about the essay (through questions and comments) in whatever ways made sense to them; only after the students had made their varied responses public did the teacher initiate a second reading, which guided the students toward a particular interpretation of the essay (the connection between the construction of specific geometric shapes and their properties). However, by grounding this successive reading in the students' initial responses, the teacher helped them both expand and refine their understanding of the specific cases being considered (i.e., construction of a circle and ellipse) and the significance of these cases for the essay as a whole ("What has the drawing of the circle have to do with the myth of the egg?"). This cycle of generating and refining meanings did not end at this point; instead, the opportunity to select one of the questions students had formulated and research it launched another cycle of meaning-making.

A final point to be made is that the readers' interpretations were not fixed by text but varied across readers and across situations. As we have shown, the meanings readers in these episodes made did not duplicate the text but arose from the confluence of readers' personal interests, knowledge, and feelings, the text, and the situation, including purposes for reading, social relations among participants, and the particular mode of communication (e.g., talking, writing ideas on cards, acting out the text, drawing the text). This was evident, for example, in Jolea's "say something" experience. Almost from the beginning, Jolea was at once puzzled and galvanized by the idea that mathematics had become intertwined with science over time, an interpretation that certainly reflected Jolea's participation in a course devoted to identifying math connections in all aspects of life. As she noted when introducing her sketch, she "didn't really look at it [the text] as really war. I looked at it as ... separating people, professions like scientists and mathematics." Where Jolea saw the text as an essay about the progressive differentiation of the roles of scientists and mathematicians, Van and Shellie saw it as an essay about mathematics as a growing discipline with new questions to be explored; moreover, none of the moral questions that were central to Char, Jolea, and RR's "say something" were represented in Van and Shellie's sketch of the text. Similarly, when students read the egg man essay using the card strategy, the variation in the questions they produced reflected a range of concerns and interests, which served as the starting point for an investigation into a concept central to understanding geometry.

5.2. Transactional Reading Strategies as Instructional Experiences

When we began the RLM project, we believed that the transactional reading strategies drawn from the reading instruction literature could offer mathematics students opportunities and structures for verbalizing, sharing, elaborating, and revising their meanings and as such show students how to approach "rich" math texts as generative readers. However, we had not anticipated the various ways the classroom teachers could adapt and combine these strategies to support her/his goals as a mathematics teacher, as the two vignettes reported in chapter 4 illustrate. And, although we had also begun with some appreciation for the way the strategies could help students make their thinking public so that their ideas could be considered further, we had not realized how powerful this experience could be. By providing readers with a flexible structure for making public even tentative and unformed ideas, these strategies taught students a way to read that valued dialogue, inquiry, and reflection. The teachers portrayed in the two vignettes, in particular, also showed us how making students' thinking public was essential to both responsive teaching and building a community of learners; the conversations that constituted whole class "sharing sessions" in these particular classrooms had a central part in enabling the teachers to build on, expand, challenge, and value students' thinking while at the same time showing students what it meant to be a member of a learning community. In what follows, we highlight these dimensions of the vignettes, which we have come to see as critical to the success of reading rich math texts generatively in mathematics classrooms.

As we have shown earlier, each of the reading strategies employed in the vignettes (say something, sketching, using cards, enacting the text) invited students to transform the author's meanings into the reader's meanings by symbolizing (e.g., talking, drawing, writing) or acting out the text. Yet, each strategy supported students' learning in a unique way. For instance, "say something" provided a structure for talking while reading. As the "Math & War" vignette clearly shows, this encouraged students to take the risks necessary to translate the text into their own words without requiring them to shape their thinking to fit a predefined task (e.g., answering teacher-prepared questions, preparing a summary, define or explain key words, concepts, or procedures). "Say something" served an additional purpose in this instructional episode in that it gave students a structure for sharing ideas with one another, something they had failed to do adequately when planning group presentations in previous experiences in the same course. The strategy thus served two goals simultaneously--learning how to make and negotiate meaning, and learning how to participate in a learning community. "Sketching" provided a natural follow-up to the "say something" strategy in this episode as it invited student to reflect on the text as a whole by translating meanings from one symbol system to another. Both pairs of students in the "Math & War" vignette used "sketching" to represent and elaborate the meanings they had produced during the say something experience. Even if it is not shown in the vignette, using drawing to think about the text was even more generative for several students, leading to the creation of original analogies and metaphors (see Siegel [1995] for examples of these sketches). Regardless of whether students used "sketching" to summarize their "say something" dialogues or to produce new insights, sharing their sketches occasioned more talk and another opportunity to generate, refine, and clarify ideas. Sharing sessions had an additional purpose in the Math connections course in that the teacher used these sharing sessions to build a learning community by helping students notice and value the kind of thinking and interactions engendered by the strategies, a point we will return to later in this section.

In the "Egg Man" vignette, the reading strategies were similarly adapted and combined in relation to the teacher's instructional goals. In the first reading, students were simply asked to read the text on their own and record their questions and comments as they read in order to encourage meaningful reading, as the "cloning an author" card strategy suggests. However, the teacher did not feel comfortable with the "mapping" activity (arranging cards in such a way as to show connections among ideas) that usually follows and on her own came up with a number of creative ways for using the cards the students had produced so that students' ideas would be heard, validated, and, in some cases, pursued further (a departure from the original "cloning an author" strategy that made us decide to refer to these variations with a new name - "using cards"). First of all, she chose to have students work as groups and sort the cards into categories before discussing the cards they regarded as most important. Although this sorting activity did not help students make connections across ideas, grouping the cards still allowed the students to experience a key feature of the cloning strategy, that is, using the cards as artifacts to support dialogue and reflection and as springboards for further thinking and inquiry. Unfortunately, there was not enough time to complete this activity, so the teacher collected all the cards (another departure from the original strategy). Looking through these cards at home, the teacher was surprised by the wealth of information they offered about what each student had made of the reading. The information she gathered led her to change her original plan as follows: she designed a homework assignment that would present a selection of the most interesting cards and invite further inquiry into some of those (to further validate the most interesting ideas generated by the class as a whole), and she decided to revisit, in more depth and as a whole class, at least some sections of the essay so that students would leave with the sense that they had understood some of the technical content embedded in the essay, especially the connection between the construction of specific geometric shapes (e.g., circles and ellipses) and their properties. The strategy she used to involve students in this reading was to have them act out what the text described, thus allowing for a more personal and concrete understanding of the mathematical concepts. This "enacting" of the text was not a strategy that came from the reading instruction literature but, instead, can be traced to common practices of mathematicians and mathematics teachers when reading difficult technical texts. In fact, it is likely that the teacher did not even think of this enactment of the text as a reading strategy at all, but as a taken-for-granted way to make sense of technical reports such as proofs--just pick up a pencil and try to reconstruct it! It is interesting to note that variations of this "enacting" strategy were observed in the other classes as well.

In the "Egg Man" vignette, too, it is important to notice the way the teacher used different strategies to achieve different goals, and how these strategies complemented one another and contributed to meaningful learning. For example, after the teacher had engaged the students in "enacting" the passage on constructing a circle, she invited discussion of the second question ("What has the drawing of the circle have to do with the myth of the egg?") so as to encourage further reflection on the text as a whole, especially the connection between the lack of information about the properties of eggs and Resch's efforts to construct an egg monument. In both vignettes, teachers adapted the reading strategies to fit their goals and combined them in ways that illustrate how a generative approach to reading, in the sense of both individual and multiple strategy use, can support the learning of technical mathematical concepts.

One pattern noted above--using the artifacts of individual strategy use (e.g., sketches, cards) as the springboard for whole class discussions, or sharing sessions--is especially important to the success of a generative approach to reading mathematics. Sharing sessions, in which students make their meanings, however tentative, public become occasions to hear and value the range of ideas and interpretations students generate as well as occasions to extend their thinking through dialogue. This was evident in the sharing session portrayed in the "Math & War" vignette, in which students explained their sketches to the rest of the class, who then had a chance to ask questions about their peers' thinking. The teacher played a critical role in making these sessions more powerful than a simple "show and tell" by asking the kinds of questions she hoped students would ask one another (e.g., she asked questions about both their interpretations and their use of the strategies), helping students look at the sketches for purposes the artists defined (e.g., she encouraged students to look for the differences between Char and Jolea's sketches once Jolea said hers had a different feeling than Char's), and extending students' thinking (e.g., she named the distinction Jolea was trying to make between mathematicians' and scientists' approach to mathematics as a difference between doing and using mathematics). A more subtle but equally significant thing the teacher did was to take the amount of time necessary for each pair of students to show their sketches, explain them, and answer questions about their experience. The strategies lose much of their power when little time is devoted to sharing interpretations, discussing the ways students used the strategies, how they felt about them, and how they might use such strategies in the future. The teacher in the "Egg Man" vignette often felt constrained by the 40 minute class period and the need to press ahead to cover the district curriculum, yet she gave whatever time was possible so that students could gain the most from the strategies; like the teacher in the "Math & War" vignette, she felt that students would quickly learn that the strategies were not much valued were sharing sessions short circuited.

In short, the sharing of meanings and perspectives served two crucial purposes in the classrooms represented in both vignettes: (1) as a springboard for dialogue; and (2) as a concrete demonstration of the social norms and values associated with being a learning community in these particular classrooms. Students in both vignettes had an opportunity to see that meanings are generated, not given, that there are multiple meanings, that these differences have value, that meanings (and readers!) grow and change; in short, they began to experience first-hand how knowledge is socially constructed.

5.3. The Challenge of Reading Difficult Texts

An explicit purpose for introducing transactional reading strategies in mathematics classrooms is to support students' reading of "rich" mathematical texts, which are often challenging for students. The first thing students commented on (or, more accurately, complained about!) was the difficulty of the texts. This is not surprising, since texts such as those used in the two vignettes are usually written with an adult audience in mind, and therefore offer different kinds of challenges for students than the truncated and decontextualized language characteristic of mathematics textbooks. In the "Math & War" vignette, for example, the students associated the introductory paragraph they read together as a class with a Congressional report! The comment, "It's a lot of bureaucratic mumble-jumble," was emblematic of their feelings about the texts they were asked to read in this portion of the class. But, as the students' experiences with the "say something" strategy demonstrate, they were usually able to get beyond this frustration and make some sense of the text. The fact that the strategy invited readers to engage in a tentative process of responding to the text, generating potential interpretations, and revising them based on the responses of other readers or the text itself provided the support they needed to turn "mumble-jumble" into an "aha!" Participating in a dialogue with others who would respond to their confusions, questions, and hunches was equally important to the success of their reading. This is vividly illustrated throughout Char, Jolea, and RR's reading of "Mathematics and War." For example, Jolea's first response to the difficult language of the text was to say she was stupid, and felt it was "cheating" to reread a text (though this allowed her to get beyond her initial confusion). Later, she confirmed the importance of dialogue when she "confessed" to RR that she would not have been able to understand this text had she been reading by herself. Van and Shellie's "question-answer" approach to the strategy is another clear example of how social interaction supported their engagement with a very sophisticated text.

The students in the second vignette also saw the "big words" as a stumbling block in reading "Adventures of an Egg Man." Though certainly not the only response they had to the text, students commented on the language of the essay during the discussion that preceded the sharing of their cards in small groups. The teacher wisely invited several students to talk about how they handled this problem and was able to help them see that one could make sense of a text without knowing every word and that word meanings could often be inferred from the text itself. Given the fact that the reading had not been presented as a case of knowledge display, the teacher was able to challenge the common misconception that a reader must understand "everything" in order to "comprehend" the text. Here again the value of talking with students about their experiences with the strategies becomes evident.

Finally, the vignettes suggest that it is important to help students stick with the strategies, despite the initial feelings of awkwardness that students are likely to feel, so they can experience the benefits that come from interacting with texts in these ways. This is why it was so important for the teacher in the "Egg Man" vignette to use the students' questions to revisit selected passages in the essay. Without a sense that the questions they had recorded on cards could be used to generate an understanding of the essay, they would have missed the opportunity to see first-hand how the strategy could support their reading and thinking. Similarly, students in the "Math & War" vignette found that saying something (even just voicing what was frustrating them) was a step toward making sense of the introductory passage. But without the kind of reflective conversation that followed the first "say something" experience, they might not have regarded this struggle to make meaning as something ordinary and expected.

In sum, the experiences reported in the two vignettes show that transactional reading strategies can provide a concrete way to support students' engagement with challenging texts, while at the same time helping them confront their feelings and conceptions of reading with the goal of demystifying this process. Though none of the texts employed in these experiences was a typical "technical math text," one could expect that many of the negative reactions students tend to have towards these texts in the context of traditional mathematics instruction might be reduced if similar supporting strategies were introduced as ways to deal constructively with these "difficult" texts; strategies that promote generative reading could supplement, if not replace, the instructional activities typically recommended in the current literature on reading mathematics, such as strategies for learning technical vocabulary and interpreting the syntax and overall organization characteristic of word problems and textbook explanations (e.g., Shuard & Rothery, 1990). Regardless of whether students read "rich math texts" or "technical math texts," it is likely that they will respond negatively, at least initially, to the idea of reading in a mathematics class. Therefore, it will be important for mathematics teachers to regard such responses as the norm, and be prepared to respond in ways that help students rethink both their expectations and their strategies for reading in mathematics classrooms.

6. What Can Students Gain from Reading Rich Math Texts Generatively?

We now turn to discuss the question which is probably uppermost in the minds of mathematics teachers and educators, that is, "What can mathematics students gain from engaging in generative reading experiences such as those illustrated in Chapter 4?" Before presenting our response to this question, however, some clarifications are in order.

The issue of determining what students gain from a given instructional strategy always presents considerable difficulty. How can we determine both short term and long term learning resulting from activities in which the strategy was used? How can we be sure that what students learned in these occasions was due to the use of the strategy and not other factors as well? How can we determine the *potential* benefits of using the strategy that may not have materialized, not because of the strategy per-se, but because of the limited extent the students engaged in it and/or how the teacher implemented it? How can we determine the *potential* benefits of using the strategy in a different instructional contexts? Given the controversial methodological issues raised by these questions, we decided in this chapter to limit our analysis of the *potential* benefits of "reading rich math texts generatively" to the following:

- *what learning opportunities were offered to students in the 18 RLM episodes* (i.e., what students *could* learn as a result of the content of the text read and the learning activities organized around it);
- *to what extent the students took up these learning opportunities in the two vignettes features in Chapter 4* (i.e., whether and how they *actually* engaged in the learning experiences planned by the teacher);
- *what the students learned from the learning experiences they engaged in these two vignettes* -- where "what was learned" is defined not only in the traditional terms of "mastery" (i.e., being able to reproduce/restate what was learned, or use it in similar circumstances) but also in terms of "exposure" (i.e., becoming aware of, gaining a new perspective on, appreciating, or coming to value particular ideas, concepts, strategies, and the like).

More specifically, in this chapter we will first of all try to carefully identify the instructional goals that the generative reading experiences developed in each of the two episodes reported in Chapter 4 were designed to support, and then document how students took up these learning opportunities and what they learned from these experiences. The results of this analysis will be organized around specific learning opportunities identified within each of the three complementary categories of instructional goals articulated in Chapter 2 -- i.e., learning about the nature of mathematics, learning mathematical content, and learning about process/learning strategies. In order to show how the gains emerging from this analysis are not unique to the two vignettes examined in-depth, we will then examine which of the learning opportunities thus identified were offered within the other 16 RLM episodes generated in our study.

6.1. In-depth Analysis of the Learning Opportunities Offered and Capitalized Upon in our Prototypical Examples

Understanding the Nature of Mathematics

Gain a better understanding of the connection of mathematics to life and other fields: Because of their content, the essays read in the two vignettes offered students the opportunity to know more about how mathematics is used in real-life. Whether they read about the role played by mathematicians in wars since ancient times, or about how a computer science professor used his mathematical knowledge and ability to build a "giant egg," the students were exposed to much more authentic and inspiring examples of applications of mathematics than the trivial word problems usually included in mathematics textbooks, thus encountering aspects of "real-life mathematical problems" many of them had not been previously aware of. For instance, the essay read in the "Egg Man" vignette makes clear how a real-life mathematical problem can take years and the concerted effort of several people in order to be "solved" (in contrast to the widespread student belief that "good" students should solve math problems in 5 minutes or less, as reported in Schoenfeld, 1992). In the "Math & War" vignette we have even more explicit evidence that the students recognized and valued new connections between mathematics and life -- as expressed for example by a student in the introductory activity ("I think I understand. It's supposed to be like what this class is about. Because it is talking about how people don't realize that math is related to other things"), as well as by Jolea and Char at several points in their conversation as well as in their sketches. Although our comments about what "counts" as learning mathematics may seem to suggest that exposure to the content of these texts alone could increase the students' awareness of the role played by mathematics in real-life -- even if they did not understand all the details reported in the essay, or discuss it in depth -- it is important to realize that the generative reading of these texts was crucial to enabling the students in both vignettes to appreciate the significance of what they read. It is especially important that several students, after initially reading the "Egg Man" essay on their own, wrote cards questioning "What has this to do with math?"

Develop a sense of the history of mathematics: Once again, the content itself of the "Math & War" essay exposed students to information about how mathematics, and specifically its use in the service of warfare, has grown over time. This issue obviously intrigued several of the students, since Jolea and Char explicitly discussed it in both their "say something" session and in their sketches, and Van and Shellie chose to depict the very theme of "change" in their sketch. Indeed, one of the main insights gained by Jolea, Char, Van, and Shellie from this reading was the realization that mathematics is a field that continues to grow -- quite a contrast to the common perception that "everything that could be discovered in mathematics has already been discovered" (Borasi, 1992). A similar opportunity to appreciate that mathematics is a growing discipline was offered by the "Egg Man" essay, as this text relates how a professional mathematician working today found it necessary to identify on his own the properties of a geometric shape that no mathematician had studied before -- a surprising event for many students (as suggested by the student-generated card "If mathematicians are so great, why don't they find a formula to build an egg?"). It is also important to realize, however, that this point and its significance may not have been fully appreciated by the students in the geometry class at this point in time, since this issue was not explicitly discussed in class.

Become aware that mathematics is the product of human activity: Although this was not a theme made explicit in either essay, students in both vignettes came to grips with some aspects of this issue. In the "Math & War" vignette, this is especially evident during Char and Jolea's "say something" session, as they discussed the social role of mathematicians (as "people") over time as well as the growing ethical and moral concerns that some individual mathematicians felt regarding the potential use made of their work for war purposes -- an issue that will be revisited in more depth later. In the second vignette, the whole story of how Resch approached the study of the egg-shape, along with the information provided about his personality and previous career, is a wonderful illustration of how mathematical results are created by real people, like Resch, and implies that the properties reported in geometry textbooks for more familiar figures, too, must have been derived by someone in the past who was interested in determining those properties (most likely, because of some applications!).

Become aware of humanistic aspects of mathematics: The "Math & War" essay explicitly tries to highlight humanistic dimensions of mathematics such as the moral and ethical issues that may arise from its use. However, the extent to which Char and Jolea were able to pick up and elaborate on the ethical dimensions of mathematics in their "say something" reading of the essay was remarkable. Although these students did not seem to have thought much about this aspect of mathematics before, their discussion of this issue was quite deep and sophisticated and added new insights and connections to what was presented in the text -- an impressive achievement even for adult readers of the same text! And, although some of their classmates may have overlooked this issue in their first reading of the text, Char and Jolea's "sharing" of their sketches provided the whole class with an additional opportunity to appreciate the meaning and importance of ethical issues in mathematics.

Gain a better understanding of the nature and scope of specific branches of mathematics[1]*:* The "Egg Man" essay as a whole could be considered a wonderful introduction to the very "essence" of classical geometry -- since it illustrates how the study of geometric figures may be important for several practical as well as mathematical applications, and identifies some of the key elements of this study, such as deciding which shapes are worth studying, defining each shape precisely, identifying characteristic properties (through the identification of what is common to many examples of the same shape), and developing methods to construct and/or reproduce those shapes. It is interesting to note that most students were not able to appreciate this message in their first reading of the essay (as illustrated by several cards questioning "What has this/an egg to do with math/geometry?"), thus showing that such an understanding of what geometry is about is usually quite foreign and not easy to grasp for middle school students. The teacher's effort to engage the students in a more meaningful, active, and supported reading of the section about the construction of circle, ellipse, and egg-shape, in the second lesson, was intended to help the

[1] This goal and the one that follows could also be considered examples of both "learning about the nature of mathematics" and "learning mathematical content." However, to avoid repetition, we have chosen to list them here only.

students make sense of this crucial point -- though it is difficult to say to what extent this ambitious goal was achieved at that point in time.

Gain a better understanding of some "metamathematical notions" (e.g., definition, proof, model): The "Egg Man" vignette offers a good example of supporting this kind of goal with respect to the specific notion of "method of construction." Not only did the essay students read deal explicitly with the issue of what it means to develop a method of construction for a given geometric figure, but the previously mentioned class activity -- where the students engaged in reproducing the methods for constructing circle and ellipse, and discussed the key connection between these methods and the characterizing properties of the figure in question -- focused on this notion and engaged the students in a much deeper thinking about the nature and role of methods of construction than is usual until (if ever!) they take an advanced course in geometry.

Learning Math Content

Since learning specific math content was not one of the goals of the "Math & War" instructional episode, this analysis will focus only on the "Egg Man" vignette.

Gain a better understanding of some "big ideas" within a math topic: This kind of learning goal overlaps somewhat with the last two categories discussed in the previous subsection. Therefore, here we will just list a few additional examples of "big ideas" about geometry that were highlighted in the "Egg Man" essay: recognizing the properties of a given shape can be very useful in applications (e.g., "virtually all forms in nature serve a function" -- p.83), and "finding the densest way to pack various geometric objects is an age-old problem in mathematics that has engendered much discussion" (p.85).

Make connections between isolated concepts/facts and across areas of mathematics: In the "Egg Man" essay, the report of how Resch approached and eventually solved the problem of building a giant egg shows connections between properties of geometric figures and tessellations that are not obvious. It is doubtful that many of the students who read the essay in the second vignette were able to fully appreciate the significance of these connections within mathematics itself, though we suggest that even their exposure to them was valuable.

Learn with understanding a specific concept/technique/fact: A good example of pursuing and achieving this kind of learning goal is provided by the class activity where the students engaged in reproducing the construction of circle and ellipse described in the "Egg Man" essay. In this occasion, the hands-on activities and the discussion that accompanied them enabled students to not only make sense of the details of the constructions described in the essay, but also gain a better understanding of the characterizing properties of circle and ellipse, and their relationship with the rigorous method of construction for each of these figures. As shown in the dialogue reported in this vignette, these notions had not been previous mastered nor understood by most students; and though no official testing of this content was done at the time, in a later unit the teacher remarked on how well the students seemed to have retained a solid understanding of circles and were able to apply it in a different context.

Become aware of some new mathematical facts, properties, and the like: The "Egg Man" essay contained a wealth of information about mathematical facts (often followed by non-technical yet effective explanations) that students had probably never encountered before. The following excerpts are just some of the many examples embedded in the essay, and well illustrate the unusual way this technical information was provided:

> In the absence of external forces, the egg would be a sphere, a shape that minimizes contact with the rest of the world. Given a certain volume of fluid, of all shapes that could contain that volume, the sphere has the smallest surface area. (Hoffman, 1988, p.83) (no further explanation given in the text)
>
> The hexagon, the square and the equilateral triangle are the only other regular polygons (straight-line figures with all sides equal and all angles equal) that can tile the plane by meeting corner to corner, but for the bee the hexagon is superior because it encloses the most area for a given perimeter. In other words, it holds the maximum amount of honey for the minimum expenditure of wax. (Hoffman, 1988, p.95) (followed by a very clear explanation using diagrams of how these are the only regular polygons that tessellate)

The teacher had made clear that students were not responsible for understanding all the technical details reported in the article and, with the exception of the circle and ellipse construction, she did not revisit any of the math content in class. Yet, some students were able, or at least tried, to make sense of this information from the text on their own (as shown by several pertinent questions reported on their cards, such as: "Are 2 identical triangles equal to a square if they fit in it?", "How do you construct an egg out of equilateral triangles and three pointed stars?", and "How can star be hexagons?"). Furthermore, we believe that the very fact of being exposed to these facts within a meaningful context should be considered positive, as it may trigger some recognition when the same students encounter these mathematical ideas more formally later in their schooling.

Learning about Process/Learning Strategies

Become familiar with specific problem solving strategies: In the "Egg Man" vignette, as the students read the real-life story of how a professional mathematician approached the complex problem of building a giant egg, they were indirectly exposed to a number of important elements in the process of solving mathematical problems. For example, the essay illustrated the importance of problem definition and its relationship with the practical constraints one is working with as well as the kind of solutions sought (i.e., trying to determine a method of construction for egg-shapes vs. figuring out what tiles could be used to build an egg-like surface) and the role played by similar but simpler problems as "stepping stones" (such as looking first at the construction of circles and ellipses to gain some ideas about how to construct an egg). There is no indication from the students' cards nor the class discussions whether students appreciated the significance of these elements; however, we consider it valuable that they were exposed to them through their reading of the "Egg Man" essay and we are aware that this dimension of the text could have been further capitalized upon by the teacher had she chosen to do so in this class. The "Math & War" vignette illustrates a quite different way in which students may learn about the process of solving problems by engaging in the generative reading of rich math texts. In this case, there is evidence that, as a result of sketches like that of Van and Shellie's (see Figure 4.3), some students acquired a new problem solving strategy -- i.e., using analogies to make sense of a complex problem. It is interesting to note that it was a student other than the authors of this sketch (Char) that showed evidence of having internalized this strategy as she used it repeatedly and effectively later in the course (see especially the "Analog & Analytic"/C7 episode) -- thus demonstrating the value of "sharing" one's sketches or, even more generally, one's interpretations of a reading, along with the process that led to it.

Acquire some specific transactional strategies to make sense of texts and use reading to support learning: Both vignettes provide evidence that, even the first time transactional reading strategies such as "say something", "sketching" and using "cards" were introduced, the students were able to engage fruitfully in them. Students in both classes continued to use all of these strategies effectively in later RLM episodes (see previous Table 3). Additional evidence of how students in the "Math Connections" course internalized and valued all the transactional strategies introduced to them can be seen in their choice of how to read texts they were expected to "teach" to the rest of the class (see RLM episodes C3-C7).

Acquire strategies to share information and insights gained from the reading of a text: Though this goal was explicitly pursued only in the "Math Connections" course, where the teacher explicitly presented it to the students as the main rationale for introducing transactional reading strategies, it is important to realize that this learning opportunity is embedded in all the reading experiences illustrated in the two vignettes because of the key role played by "sharing" and discussing the meanings created from the individual generative readings of a text. It is also worth noting that by offering vehicles that improve mathematics students' ability to share information, generative reading experiences such as those illustrated in our two vignettes can also contribute to the more general goals of enabling students to communicate mathematically (e.g., NCTM, 1989, p.5) and to effectively become a community of learners.

Engage in and value "learning how to learn": While the previous goals about "process" focused on students' increased awareness and proficiency in the use of some specific problem solving or reading strategies, this category focuses on those learning opportunities, offered to and capitalized upon by students in the two vignettes, that had more to do with metacognition. In particular, we would like to characterize as cases of "learning how to learn" the learning opportunities that were the result of more intentional and explicit reflections on what students did to learn and their significance for future applications. Note that this goes beyond acquiring problem solving or reading strategies and may also go beyond reading and/or math learning specifically, as it could provide students with more general insights into the process of learning. The considerable class time and attention devoted in both vignettes to discussing the process the students had gone through is strong evidence that "learning how to learn" was deemed important by both teachers in these classes. Furthermore, in the "Math & War" vignette, the students' explicit comments on the value of the transactional reading strategies stand as evidence that many of the students in this class were able to achieve this goal to a remarkable extent. In particular, it is worth highlighting how Jolea came to explicitly appreciate the value of stopping to work through a reading to make sense of it, which is the essence of the "say something" strategy ("I don't think I'd be able to read from a book [like this] without reading and then talking about it") and was even able to extend it beyond the strategy itself as she expressed her belief that she could use some elements of the say something even when reading an article by herself. In the same vignette, Char was also able to articulate how sketching provided her with a concrete vehicle to "pull things together" as well as to record and communicate the sense she had made of an article, and several other students in the class commented on the value of sharing and hearing others' approaches in the class discussion that followed the sharing of the sketches. In both vignettes we have also examples of how the students came to insights about reading that went beyond the use of the specific transactional strategies introduced by the teacher, such as Jolea's realization that a reading may not merely answer questions but also raise some new ones and that this could be valuable per-se, and the students' appreciation in the "Egg Man" vignette that one does not have to understand every word and detail of a text in order to get something worthwhile out of it, reached as a result of an explicit classroom discussion on this topic.

6.2. A Look at the Learning Opportunities Offered by All 18 RLM Episodes

The list of learning opportunities offered by the generative reading of rich math texts identified by the previous analysis, although already very rich, it is by no means exhaustive (we remind the reader that it was generated by the analysis of only two vignettes). Yet, by itself it suggests that the proposed approach to reading mathematics can support a variety of learning goals that, though deemed important by the mathematics education community, have so far been neglected (if not totally ignored!) in traditional mathematics instruction.

Table 6.1 shows how these learning opportunities were not unique to the two vignettes examined in more depth; rather, most of them were offered by several RLM episodes developed in quite different instructional settings. (Note that in this table, as well as the ones that follow, each RLM episode has been identified by its short title and code -- see Appendix for full titles and descriptions; both the codes and the thicker vertical lines were used to help the reader identify the different classrooms where these episodes developed.)

INSERT TABLE 6.1 ABOUT HERE

It is especially significant to note that, as documented in Table 6.1, *every one* of our 18 RLM episodes provided students with opportunities to learn about various aspects of mathematics as a discipline -- something which is currently missing from almost any secondary math curriculum. Similarly, Table 6.1 shows how most RLM episodes offered ample opportunities for students to learn new learning strategies and ways to voice their thinking. Opportunities to "learn how to learn," instead, were only offered in those environments and/or episodes where sufficient class time and explicit attention was devoted to a discussion of the process (as happened throughout the "Math Connections" course and in the two major RLM episodes developed in the geometry unit). Finally, the data reported in Table 6.1 also suggest that, even though the learning of specific math content was not one of our main reasons for suggesting the introduction of "reading rich math texts generatively" in mathematics instruction, this way of reading mathematics can support this kind of instructional goal as well, provided that appropriate texts are chosen.

It is also important to highlight that our analysis has shown that, though exposure to richer texts alone may provide students with some of the learning opportunities discussed above, *how* the text is read also considerably affects what the students "get" from reading it. For example, the use of transactional reading strategies, especially when accompanied by explicit reflection on the process, seems to be quite conducive to providing students with the opportunity to focus on "learning how to learn," because they offer structures for making students' thinking public and in doing so value it. Similarly, the kind of active and social engagement with the text supported by most generative ways of reading can provide valuable structures that support students in constructing for themselves meaningful interpretations of the technical content presented in the text -- thus contributing to the development of the kind of understanding advocated by mathematics education researchers working from a constructivist perspective.

Altogether, the findings reported and discussed in this section provide empirical evidence of the many complementary ways in which "reading rich math texts generatively" can support students' learning of mathematics. This confirms and further articulates our original belief in the potential value of this way of reading mathematics for mathematics instruction.

7. Ways of Reading Generatively in the Mathematics Classroom

In this chapter we report our major findings with respect to the research question "What, why, and how did the students read?" in our RLM episodes, with the goal of highlighting the variety of ways reading can be used productively in mathematics instruction. The brief descriptions of the 18 RLM episodes reported in the Appendix, especially when complemented by the two detailed vignettes reported in Chapter 4, already provide evidence that reading generatively can take on many different forms in the mathematics classroom. A systematic analysis of "what, why, and how" students read in each of these episodes not only confirmed the existence of many different kinds of texts, reading strategies, and purposes for reading that mathematics teachers could consider when planning lessons, but also challenged somewhat our original conception of "reading to learn mathematics."

Tables 7.1 and 7.2 report the results of our analysis of what was read, why, and how in the "Fig Leaf 1"/C1 and "Egg Man"/G2 RLM episodes from which the vignette reported Chapter 4 were constructed. Similar tables were created for all the other 16 RLM episodes, though they could not be included due to space constraints.

[INSERT TABLE 7.1 AND 7.2 ABOUT HERE]

7.1. Expanding Our Original View of "Reading to Learn Mathematics"

The reading experiences identified in Tables 7.1 and 7.2 illustrate a number of interesting uses of reading that required us to revisit our original notion of "reading to learn mathematics." Despite the fact that each of our RLM episodes was identified on the basis of involving the generative reading of a "rich" text, our "what/why/how" tables also revealed other forms of reading mathematics as well as close interactions between reading and other modes of learning. More specifically:

- Most RLM episodes involved, besides the key "rich text" around which the learning experiences were constructed, the *use of multiple texts* (e.g., sketches and newsprint where students' comments were recorded in the "Math & War" vignette, directions for assignments and index cards written by the students in the "Egg Man" vignette). It is interesting to note that several of these texts were not necessarily what we initially thought of as "rich math texts." Regardless of their "richness," the interaction among these various texts was a crucial element in all RLM episodes of a certain complexity, such as those illustrated in the two vignettes, a finding which resonates with the recent interest in intertextuality among reading researchers noted in Chapter 2.

- *Not all the texts used in an RLM episode were read "generatively"* (e.g., in the "Math & War" vignette the students just glanced at the newsprint as it was created; in the "Egg Man" vignette, the written directions provided by the teacher were read by the students with the mere goal of comprehending the task they were supposed to perform, and even the "Adventures of an Egg Man" essay was just "skimmed" when the students were asked to identify names and uses of geometric figures within it for homework). It is important, however, to appreciate that in all of these cases it would not have made sense to read the text in any other way, given the purpose of that reading event within the overall instructional episode; we would therefore argue that the kind of reading required in these situations contributed in distinct ways to the learning experience the students were engaging in.
- In many RLM episodes, it would be *artificial to try to separate "reading activities" per-se from other literacy activities*, since the generative reading of a text often involved some writing (e.g., writing cards in the "Egg Man" vignette) and small group and/or class discussions. Indeed, it seems to us that reading, writing, and talking become more connected and "transparent" when teachers orchestrate genuine learning experiences that treat these forms of literacy as important modes of learning, as happened in our units.
- Similarly, it was often *difficult to separate the generative reading of a specific text from other mathematics activities orchestrated around it* (think, for example, of the hands-on activities the students engaged in to identify the characteristic properties of an ellipse that had not been mentioned in the "Adventure of an Egg Man" essay, in their final reading of that text in the second vignette). Indeed, we observed that in some occasions (as best illustrated in the "Lake

Problem"/G1 and the "Analog & Analytic" /C7 episodes) the generative reading of a rich math text could act as the catalyst for inquiry that then takes on a life of its own.

These last two points remind us of the need to reconceive what it means to 'read' when taking a transactional perspective on reading, as any generative reading is likely to involve much more than simply "going through a text to decode the symbols in it." Furthermore, they seem to support a-posteriori our decision to choose *RLM episodes* as a unit of analysis for this specific study. Given our ultimate goal of developing an approach to reading mathematics that would support the new goals for school mathematics, looking at what happens in an instructional episode where rich math texts and a generative use of reading are employed would indeed seem more likely to yield ideas for meaningful and integrated learning experiences for mathematics students, than a narrow definition of 'reading' and a focus on individual reading events/experiences.

Even more importantly, the finding that several (and not necessarily "rich") texts were also read in non-generative ways in most of our RLM episodes suggests that reading practices other than "reading rich math texts generatively" could contribute to students' learning in important ways and, thus, should not be dismissed in an analysis of how reading mathematics can support mathematics instruction. Elsewhere (Siegel & Fonzi, 1995) we have begun to analyze these complementary kinds of "reading practices" in the context of the "Math Connections" course. At the same time, we believe that this realization does not undermine the explicit study of "reading rich math texts generatively" undertaken in this article, provided that it is recognized that this is not the *only* way reading could be used productively in mathematics instruction. Rather, we believe that a generative approach to reading mathematics merits special attention, as it is currently unfamiliar to mathematics educators, yet can contribute something unique to the mathematics education literature, both as an instructional strategy that supports the new goals for school mathematics, and as a catalyst for helping mathematics teachers and students become aware of the potential of reading to support mathematics learning in many other forms as well.

7.2. Variations Within "Reading Rich Math Texts Generatively"

Even when considering only reading experiences involving the "generative reading of rich math texts," the "what/why/how" tables created for our RLM episodes revealed a considerable variety in terms of texts read, reading strategies employed, and learning goals promoted by this kind of reading mathematics. Table 7.3 summarizes this information for all the texts read generatively throughout our 18 RLM episodes. In what follows, we will briefly comment on what this analysis contributed to our understanding of what, why, and how texts can be read generatively in mathematics classes.

[INSERT TABLE 3 ABOUT HERE]

What students read: The texts read in our 18 RLM episodes present an interesting variety with respect to both format and content. Although the great majority of the texts read generatively in our five "units" were essays, we have also examples involving magazine and newspaper articles, excerpts from textbooks and novels, and even teacher-generated diagrams and lists of questions. The content of these texts also spanned from non-technical explanations of math concepts and/or topics, to historical accounts and/or considerations, information about applications of mathematics in various fields, accounts of the process followed to solve complex problems, and even background information about a real-life situation that the students themselves were trying to use mathematically (e.g., the articles read in the "Nutrition"/L1 episode, intended to provide the students with lists of food categories they could then manipulate using logical connectives to create well-balanced meals). With a few exceptions (such as the excerpts read from the textbook in the "Lake Problem"/G1 and "Careers"/C8 episodes), none of these texts would have found its way in a traditional mathematics classroom. Yet, both the content and the format of these texts contributed new dimensions to students' learning in our units by dealing with aspects of mathematics usually neglected in traditional schooling, as well illustrated by our two vignettes and further supported by the information on the other RLM episodes provided in the brief descriptions reported in the Appendix. At the same time, the inclusion of texts such as diagrams (see "Area Diagrams"/G3 episode) or lists of questions (see "Questions"/p1 episode) in our list also suggests that what makes a text "rich" may have more to do with the way it is used within a learning activity, than its content or format per-se (cf. Siegel & Fonzi, 1995).

Why students read these texts: The experiences organized around the reading of all the rich texts identified above were intended to support the learning of many important mathematical concepts and topics, and often engaged students in considering issues related to the nature of mathematics as a discipline and the process of doing mathematics in a variety of ways. Evidence in support of this claim has already been provided in the previous chapter (in particular, see Table 6.1 for a summary of the key learning opportunities offered by each RLM episode). Further information about the variety of mathematical topics as well as issues about the nature of mathematics and its learning addressed in each of the RLM episodes is provided by the "WHY" column in Table 7.3.

How students read these texts: The "HOW" column in Table 7.3 shows that "reading generatively" took on a number of different forms across the 18 RLM episodes, while building in most cases on one or more of the four major "transactional reading strategies" identified in the introduction (i.e., "say something," "sketching," "using cards," and "enacting"). In all of these situations, the students seemed quite able to use these reading strategies without difficulty and most often in very productive ways (though not always to the extent illustrated by the students in the two vignettes reported in Chapter 4). What is most interesting, however, is how individual teachers made these strategies "their own" by interpreting, adapting, and modifying them for specific instructional episodes, thus demonstrating in practice the "flexibility" of these strategies. Table 7.4 identifies all these variations and their occurrence in the 18 RLM episodes.

INSERT TABLE 7.4 ABOUT HERE

To avoid repetitions, here we would like to limit our comments to a few points that can highlight the information provided by Table 7.4. First of all, we would like to point out the many different ways in which cards were used and the spontaneous development by several teachers of variations of the "enacting" strategy that seemed particularly appropriate to reading math-related texts. Although the "sketching" was used only in two occasions, it is important to note that it inspired the activity of translating a "text without words" (see "Area Diagrams"/G3) into words or mathematical symbols - a strategy that, like "sketching", could be thought of as another example of "transmediation" (i.e., the transformation of a given text into a different symbol system). It is also important to note that a combination of reading strategies (and/or variations within the same strategy) was often employed in the same RLM episode for the reading of the same text (as revealed by looking along the columns in Table 7.4). Finally, "sharing," -- the opportunity for individual students to communicate and discuss the meaning they made from their reading of a specific text -- could also take on different forms, as indicated by the variations shown in Table 7.4 under this subheading for each main strategy, especially in conjunction with the use made of students' writing in the "using cards" strategy; indeed, some of the teachers in our project paid considerable attention to providing students with vehicles that would encourage and support productive sharing.

In sum, the previous analysis not only confirms our original belief in the possibility of successfully introducing the reading of texts other than textbooks and worksheets using transactional reading strategies in a variety of mathematics classes, but also illustrates how "reading rich math texts generatively" can take on many forms and variations. And, even if it should be clear that the texts, strategies, and learning goals identified in this section are only a subset of what could be possibly considered by mathematics teachers (since the categories used in our tables were derived exclusively from the compilation of what was observed in our 18 RLM episodes), they already suggest a vast array of possibilities that could considerably expand the current repertoire of mathematics teachers.

8. Orchestration of Reading Experiences in Mathematics Classroom

The reading experiences developed in our 18 RLM episodes suggest that indeed students can productively engage in the "generative reading of rich math texts" in the context of mathematics instruction. At the same time, as already argued earlier in chapter 5, it is important to appreciate that a good measure of this success depends on how each teacher orchestrates these reading experiences in his/her classroom. In this section, we will identify and discuss some of the elements that we believe can most affect the implementation of successful generative reading experiences in the context of mathematics instruction based on the analysis of our RLM episodes.

8.1. Establishing a Supportive Learning Environment

First of all, both the "success" of experiences involving the "reading of rich math texts generatively," and the extent to which these experiences contributed to realizing the goals for school mathematics called for by many constituencies today (as noted in Chapter 2) seem to depend to a large degree on having established in the classroom social norms, values, and an overall learning environment that support these goals as well as the proposed approach to reading mathematics more specifically. For example, the instructional decisions, actions, and discourse portrayed in the two vignettes reported in Chapter 4 and further discussed in Chapter 5, indicate that the two teachers who orchestrated these learning experiences shared assumptions such as: learning involves struggle, and such a struggle is valuable; working with peers to clarify and further expand one's thinking is worthwhile; students' voices should be heard and valued in the classroom; "learning how to learn" should be an important goal of school mathematics. Moreover, these values informed all of their teaching, not just the uses of reading introduced in their classrooms. We believe that if teachers do not share these values, which are at the very core of any transactional reading strategy, the effectiveness of generative reading experiences is considerably reduced, as the students may (correctly!) perceive them as an "isolated" aspect of the course and, therefore, not worth their effort -- as was the case to some extent in the other two classrooms. On the other hand, it is worth noting that a consistent and integrated use of generative reading experiences could contribute much to shaping a different kind of learning environment, especially a "learning community" (Siegel & Fonzi, 1995), since by their very nature transactional reading strategies offer a vehicle to encourage students' participation and voice in the mathematics classroom.

As suggested above, students may not seriously engage in generative reading experiences that are included in a mathematics course unless they see that the teacher considers these experiences to play an important and worthwhile role in their learning. An analysis of the teachers' instructional choices and actions in our two prototypical examples suggests some concrete ways in which teachers can show their students that they value the generative reading of rich math texts:

- by making students explicitly aware of the role these reading experiences can play in their learning of mathematics -- since students are not used to reading in a mathematics class and may initially be skeptical about its value (as documented even in the "Egg Man" vignette!); as illustrated by the teachers in both vignettes, this can be done by explicitly articulating the rationale for each reading task the students are expected to engage in, especially when a new transactional strategy is introduced, and by involving the students afterward in an analysis of the process (i.e., how they used the strategy and what they got out of it); this reflection can take various forms such as a pre-planned class discussion, focused journal entries, and/or just encouraging and capitalizing on students' spontaneous comments;
- by devoting the class time needed to do the actual reading of the text in class (as illustrated in the "Math & War" vignette), pursue follow-up activities that center on making sense of the text (as in the case of the "Egg Man" vignette, where the first reading of the essay was assigned for homework, but then two class periods were devoted to activities developed to help students understand this text), and to "share" and discussion the process.

INSERT TABLE 8.1

Table 8.1 shows how the teacher in the "Math Connections" course used all of these strategies consistently in the RLM episodes she created, as did the teacher in the geometry unit for the more complex RLM episodes developed in her class. The RLM episodes developed in the other two classes show instead a quite different pattern, due probably to a variety of factors including the constraints presented in these situations by a final state-wide exam and a more traditional school environment, as well as these teachers' prior teaching experience and pedagogical beliefs. The data reported in Table 8.1, however, seem to us indicative of the different values informing these classes which, in turn, led to RLM episodes that were somewhat more constrained and less rich than those developed in the two other instructional contexts, though we believe that they still offered students some valuable learning opportunities and experiences.

8.2. Teacher's Choice and Adaptation of Specific Strategies

A main point made throughout this article, on both theoretical and empirical grounds, is that "reading rich math texts generatively" can take a variety of forms and, more specifically, that transactional reading strategies should not be considered as rigid techniques, but rather as heuristics that need to be adapted and modified to meet the unique needs of each instructional situation. This point is well illustrated by the analysis of the teachers' rationale for key instructional decisions made in the two vignettes, as noted in the discussion of these vignettes presented in Chapter 5. Indeed, we believe that a good decision about how to read a given text can only be made on the basis of the teacher's instructional goals for a particular reading experience as well as for the larger instructional episode in which it is embedded. For example, in the case of the "Math & War" vignette it is important to remember that the decision to read the *same texts* using three major transactional strategies (i.e., "say something", "sketching" and, later in RLM episode C2, also the "using cards" strategy) was motivated by the ultimate goal of helping the students in the "Math Connections" course develop ways to "share their thinking with others" and to make sense of information provided by texts -- weaknesses that the students had identified in themselves while reflecting on their performance in a previous group project. Similarly, "using cards" seemed a natural choice to the teacher in the "Egg Man" vignette, since in this case the mere length of the essay, combined with the fact that she was not expecting the students to understand most of the technical details it contained, made its reading in class impractical. The teacher's decision to use this strategy was thus a result of these considerations as well as her desire to offer her students a vehicle to help them stay focused and to record some of the thoughts and questions generated by their individual reading so that they could be later shared with the rest of the class.

Table 7.4, discussed in the previous chapter, has already shown how the four teachers in our project implemented several variations of the four main transactional reading strategies identified in the introduction -- i.e., "say something," "sketching," "using cards" and "enacting" strategies. It is important to note that, with the exception of some variations within the "enacting" category, all these strategies were unknown to any of these teachers prior to their participation in the RLM project. Experiencing "as learners" at least one variation within each main category in the professional development seminar was crucial to the teachers' use of these strategies as it gave them an understanding of how these strategies could play out in the context of mathematics instruction and some sense of how they could use them productively in their own mathematics classes. Yet, these demonstrations of the strategies did not in any way determine or limit teachers' choices, as shown by the different strategies teachers selected to use in specific RLM episodes and the adaptations they made; on occasion, they even developed new strategies (see Table 7.4). For example, sketching and the mapping of cards were used only in the "Math Connections" course, despite the fact that both strategies had been presented in the professional development seminar. Some teachers also developed new variations of "enacting" on their own.

While the presence of these variations are important for us, as they indicate that the teachers understood the intended spirit of the transactional reading strategies that was conveyed in the professional development seminar and were able to make them "their own," it is also important to notice that modifying a reading strategy as originally presented did not always have a positive outcome. Rather, in a few cases it seems to us that the variation resulted from the teacher's (or students') failure to fully appreciate the reasons for using important components of the original strategy and, as such, diminished the strategy's potential to support meaning-making and learning. This was especially the case for the "using cards" strategy. "Mapping" the cards students produce while reading a text on their own is a crucial element of "cloning an author" (Harste & Short, with Burke, 1988), as it requires the readers to go beyond a superficial reading of the text and grapple with the relationships among key ideas presented, as well as to share and negotiate different possible interpretations of the text. Yet, three of the four teachers who used "cards" in their classes, and all but one of the students in the "Math Connections" course who chose to use this strategy to make sense and then "teach" a text of their choice to the rest of the class (see C3 - C6 episodes), did not do any mapping of the cards. Though in some of these cases the students still engaged in valuable alternative uses of the cards (as illustrated in the "Egg Man" vignette), we also have examples where the use made of student-generated cards were rather minimal (see in particular the description of the uses of cards made in the "Platonic Solids"/G5, "Large Numbers"/P2, "News Article"/P3 and "Questions"/p1 episodes in the Appendix). In these cases, although *writing* the cards per-se was beneficial to the students as it often helped them make sense of the text they had to read, the limited *use* made of the cards considerably reduced the opportunities to support thinking and talking that the "using cards" strategy can offer as opposed to simply "taking notes." This was especially the case when the cards were collected (as is often done with written work in mathematics classes) and the students were then asked to discuss the text *without the support provided by the cards as artifacts* (as happened to some extent in the "Large Numbers"/P2 and "News Articles"/P3 episodes).

8.3. Introducing a Reading Strategy for the First Time

Because the generative reading experiences we are proposing are so different from the way students are used to reading (especially in the context of school mathematics), and may therefore come up against some of the beliefs students have about reading in general and reading mathematics in particular, we believe it is very important that teachers pay explicit attention to how a transactional reading strategy is introduced for the first time. For this reason, when we selected which RLM episodes to present in our vignettes, one of the criteria for choosing episodes was whether a new strategy had been introduced (e.g., students had their first experiences with both "say something" and "sketching" strategies in the "Math & War" vignette, and the "using cards" strategy in the "Egg Man" vignette).

Through an analysis of how these two teachers introduced a new transactional strategy in these two contexts, we identified the following key elements:

- the importance of explicitly discussing the rationale for that strategy and its potential value -- to be done by the teacher when introducing the reading task, and in a class discussion after the students have had an opportunity to personally experience the strategy;
- the importance of providing the students with a good image of what the strategy would "look like" -- which could take different (and sometimes complementary) forms such as: "modeling" the strategy (as done with "say something" in the "Math & War" vignette); providing some carefully worded and detailed verbal directions (as done for some variations of "using cards" in the "Egg Man" vignette); making an explicit connection with another reading strategy the students have previously used and are therefore familiar with (as when in

the "Egg Man" vignette the first use of the "card" strategy -- i.e., recording key ideas, thoughts, questions while reading -- was presented to the students as "doing a say something without a partner").

The way "sketching" was introduced in the "Math & War" vignette presents an interesting exception to this idea, since in this case the students were simply given the open-ended and rather ambiguous (though carefully worded!) task of "drawing what they learned about mathematics from the text" they had just read. The reasoning behind this decision not to model or offer precise directions for "sketching" was based on the fact that student often feel quite inhibited about drawing and are often looking for a way to resolve the ambiguity inherent in translating from one symbol system to another. And yet this ambiguity is the source of the generative power of sketching so it is not necessarily in the students' best interest to clear up this ambiguity for her/him. However, it is still important that students have an opportunity to share and reflect on both the process and product of sketching so that they can better understand the range of ways that this strategy could be approached as well as the learning potential it offers.

Table 8.2 reports which of the approaches identified above was used to introduce a new reading strategy in a class (note that, in this Table, each column corresponds to the *introduction of a new reading strategy* -- or a new major variation of a familiar one -- in each of the four classrooms; thus, each column has been characterized by the name of the strategy/variation introduced as well as the code of the RLM episode in which this event occurred).

INSERT TABLE 8.2 HERE

The data reported in Table 8.2 show a widespread use of at least one of these approaches and more often a combination of them. There are only two notable exceptions to this pattern, in which none of the approaches identified earlier were used when a new strategy was introduced to the students. The first occurred when the teacher in the geometry unit engaged her students in a number of mathematical activities (e.g., developing their own solutions to a problem posed in the text *before* reading the solutions proposed by the authors -- "Lake Problem"/G1). Despite the fact that this was an "unusual" though generative way to read a text, it seemed to come naturally to this teacher. Indeed, it is our impression (based on field-notes from both classroom observations and planning meetings) that she did not even think of it as a "transactional reading strategy" -- hence her failure to even perceive the need to introduce it and explicitly discuss it with the students, as she instead did with both the "say something" and "using cards" strategy. The other exception dealt instead with the introduction of a variation of the "card" strategy in the logic unit ("Nutrition"/L1 episode). In this case, the teacher was aware that he was introducing a new strategy and had provided written directions to the students about what to do with the cards; however, these directions (especially since they were not accompanied by a discussion of the nature and rationale of the strategy) were not as "detailed" as we thought necessary to helping the students develop a good image of what they were expected to do and why.

The occurrence of particular ways for introducing strategies, reported in Table 8.2, also show some interesting patterns. The decision to discuss the rationale for the strategy explicitly (before and/or after the reading experience) seems connected to the teacher's overall teaching approach, as it was used consistently in the "Math Connections" course, occasionally by the teacher in the geometry unit, and not at all by the other two teachers. All teachers seem instead to have paid attention to providing their students with a good image of how the new strategy should play out (with the exceptions noted above). Their choice of how to do so, however, seemed to depend mostly on the nature of the strategy introduced and/or the students' prior experiences with other transactional reading strategies. For example, it is interesting to note that the "say something" strategy was introduced in all four classes by means of modeling -- perhaps because of the unusual approach to reading it involves and the fact that in all cases it represented the students' first encounter with transactional reading strategies; instead, with one exception involving modeling (episode C2), any variation of the "card" strategy was either simply introduced as any other learning task in that class, or through detailed instructions and/or by making connections to the "say something" strategy.

9. Conclusions

Taken as a whole, this monograph has tried to articulate what it means to "read rich math texts generatively" in theory and in practice, and has examined several critical dimensions of the teaching/learning process that teachers would want to consider when planning such reading experiences. Here we want to highlight the most important findings and consider their significance for research and practice. First of all, the vignettes have shown that "reading rich math texts generatively" does not mean separating reading from other meaningful mathematical activities or using it as a way to introduce or finish off traditional instructional units on various topics. If this approach to reading offered little more than a curricular frill, something analogous to extraneous information in word problems or the sidebars on careers and applications one finds in mathematics textbooks, it would be difficult to convince teachers that it is worth the considerable thought and effort required. However, the vignettes presented in Chapter 4 clearly show that a generative approach to reading mathematics can make a significant contribution to the learning of mathematics when fully integrated into ongoing instructional activities. Despite all the variation in instructional purposes, the kind of texts that can be thought of as "rich" mathematical texts, and the strategies for reading them, the vignettes indicate that what distinguishes a generative approach to reading mathematics is the understanding that meanings grow and change as learners act on and act out texts and that the artifacts of these transformations enable learners to make their thinking public. As we have argued throughout the monograph, this shift toward public negotiations of texts--the process of making and remaking meanings--is central to a generative approach to reading mathematics. What is significant about these reading experiences is how they create rich opportunities for learning. Among the opportunities documented through the analysis of all 18 RLM episodes were opportunities to learn mathematical content (including technical facts, concepts, and procedures), learn about the nature of mathematics (including awareness of different branches of mathematics, "big ideas" about particular mathematical topics, humanistic aspects of mathematics, metamathematical concepts), and learning about processes and strategies (including strategies for problem solving, making sense of texts, sharing one's thinking with others, and learning how to learn). Although some educators might wish to argue that the only kind of reading that has value for learning mathematics is the kind that is distinctive to word problems and textbooks, we believe the analysis of learning opportunities offered by the RLM episodes, provides support for our contention that it is unnecessarily limiting to reduce reading in a mathematics class to the kind of reading thought to be distinctively mathematical in nature. These learning opportunities can also be taken as evidence of the relevance of instructional strategies developed by reading education researchers for mathematics instruction, a point that confirms the value and need for interdisciplinary instructional research.

It is important to notice that integrating reading into classroom learning activities created instructional episodes that were diverse and complex. Although we had never thought of "reading rich math texts generatively" as a monolithic approach but one that varied in relation to the instructional goals the teacher was trying to achieve, this sense became clearer and more significant as a result of constructing the two vignettes presented in Chapter 4. These vignettes illustrate quite clearly that "reading rich math texts generatively" looked quite different depending on the teacher's instructional purposes, which, in turn, influenced (but did not determine) what was read or how it was read. Though we believe that the teacher's instructional purposes guide the selection of "rich" texts and strategies for reading them, this does not mean that deciding what and how to read is a straightforward or trivial decision. The question of what and how students read in a mathematics classroom thus remains vital to the success of generative reading in mathematics instruction, as teachers may not appreciate the fact that there is more to read than word problems and explanations of concepts and procedures in textbooks. Recall, for example, that in both vignettes as well as the other 16 RLM episodes we developed, students read a range of materials, including essays, excerpts from novels, newspaper and magazine articles, and other texts consisting of figures or directions. The choice of text was never made without considerable reflection on the instructional purpose it would serve and the kind of reading strategy that would promote generative thinking and meaningful learning. For this reason, it is important that mathematics teachers become aware of the kinds of transactional strategies described in this article as well as make explicit their own intuitive strategies for reading mathematics that may be useful to students, such as the "enacting" strategy the teacher spontaneously invoked in the "Egg Man" vignette. Though for some mathematics teachers the mere act of reading is an innovative instructional practice (Lehmann, 1993), the data illustrate that developing an awareness of a range of transactional reading strategies enables teachers (and ultimately their students) to build a repertoire from which particular strategies can be selected, adapted, and combined to achieve their desired instructional goals. Teachers will be disappointed, however, if they expect the strategies to help students reproduce the text and arrive at what they regard as the "correct" interpretation of the text. The vignettes show, instead, that the strategies served as flexible heuristics for thinking and meaning-making, that is, ways to position and support students as active readers/learners. By providing structures for transforming author's meanings into readers' meanings and then making those meanings, feelings, questions, and processes public through "sharing sessions," the strategies enabled students to construct a personal understanding of the text while at the same time serving as a springboard for further dialogue and inquiry. These "sharing sessions" not only provided an occasion for negotiating meanings but for reflecting explicitly on students' learning processes and for learning how to act like a member of a learning community, which meant, among other things, learning to participate in a dialogue, to value and learn from the ideas and experiences of others', to rethink one's own ideas, to see difference as a generative resource, and to appreciate the complexity and power of socially constructing knowledge. These reading strategies, thus, have the potential to contribute more to meaningful mathematics instruction than merely as supports for comprehension of "rich" mathematical texts.

The episodes analyzed in this study were not only complex because of the interrelationship of what, how, and why students read, but because the transactional reading strategies required carefully thought out ways to introduce them, reflect upon them, and use them to support the kind of sharing sessions that further students' thinking and learning. As the vignettes and analyses of our prototypical examples indicate, teachers had to think not only about which "rich" texts might extend students' understanding of the concepts being studied and what kind of reading strategy might support a generative reading of that text, but they also had to consider how to orchestrate the actual use of the strategy in relation to the instructional purposes so that students would understand why the particular strategy might be of use to them, get a sense of what the strategy involved (e.g., reading and talking, drawing, writing comments or questions on cards, etc.), extend their thinking either through sharing artifacts that were produced or other related activities, and, come to value the strategies. This pedagogical work required more texts, more reading, and more talk than the application of a single reading strategy to a single "rich" texts might suggest, but without the additional texts, reading, and talk, it is unlikely that the students will experience the benefits the strategies offer and hence fail to incorporate them into their repertoire of learning strategies. This is especially important given the fact that students in the classrooms where we worked often felt that reading did not belong in a mathematics class and found the texts hard to read. Providing class time for students to try out the strategy and consciously consider their own experiences and the experiences of others is therefore an essential aspect of a generative approach to reading mathematics. This conclusion demonstrates the significance of what we originally described as a "compatible curricular framework," and what we have come to regard as a set of instructional values and social norms grounded in social construction of meaning and knowledge. Our analysis of all RLM episodes shows that when teachers do not value students' voices (i.e., their meanings, feelings, questions, processes, and even complaints), they tend not value and hence not allocate the time needed to make the strategies a meaningful part of classroom life. Though more research on what it means in practice to "read rich math texts generatively" is certainly needed, our hypothesis is that in the absence of these shared values, the contribution these strategies can make to students' mathematical learning is significantly diminished.

Finally, we think it is important to notice the extensive professional development activities that led to these RLM episodes. Recall that the four teachers with whom we collaborated were originally part of a semester-long professional development seminar in which they experienced, as learners, all of the strategies discussed in this article, and that, in the school year that followed, they had opportunities to adapt, combine, and invent their own strategies in the context of planning instructional experiences for their students. Though we have not done a systematic analysis of the relationship between the professional development seminar and the teachers' choices when planning their own instructional units, we believe this would be a fruitful question to study and one that would offer insights into the process of instructional change and have implications for teacher education. However important these experiences might have been to the teachers' understanding and use of this approach to reading mathematics, it should be noted that the teachers experienced a less formal, but quite extensive interaction with the researchers during the year devoted to collaborative action research, as noted in Chapter 3. Our sense is that these interactions, which included planning units collaboratively, documenting and participating in the class periods throughout the entire unit, conversations before and after the lesson, and regular meetings of the entire team at the school site, enabled the teachers and researchers alike to appreciate more fully what it meant to "read rich math texts generatively" in practice. For several teachers in particular, seeing the kinds of learning opportunities created by these reading experiences gave them the encouragement needed to incorporate the strategies in their instructional repertoire and use them without the collaboration of the researchers.

In conclusion, the research reported in this article suggests that putting a generative approach to reading mathematics into practice involves far more than becoming aware of a set of transactional reading strategies and identifying a list of relevant "rich" mathematical texts. It requires, instead, opportunities for teachers to experience the learning potential of this approach first-hand as well as many opportunities to engage in dialogue, reflection, and inquiry into the possibilities of reading in their own mathematics class. These observations lead us to hypothesize that just as a generative approach to reading mathematics requires considerable support if students are to tap its potential, teachers also need substantial support to see that reading can be a generative mode of learning in their classrooms and that it is worth the extra effort initially required to make it so. Reading rich math texts generatively is not the only way to think about reading mathematics, as we have argued elsewhere (Siegel & Fonzi, 1995). However, if teachers and students can experience the power of this approach to reading mathematics, it may contribute something unique to the reinvention of teaching and learning in mathematics classrooms.

APPENDIX:

SUMMARY OF "READING TO LEARN MATHEMATICS" EPISODES

(NOTE: In the text, individual episodes have been referred to by the short title and/or code preceding each full title; the codes are intended to identify the "unit" the RLM episode came from and its chronological position within that unit.)

Episodes from a geometry unit taught in a rural/suburban middle school

G1. (Lake Problem) Using geometry to find the distance across a lake *(about 6 40-minute class periods)*

This episode developed around a 6 page essay, "Elementary Geometry" (Campbell, 1976), which focused on the origins of geometry and the use of various geometry properties to find distances that can not be measured directly. The students read the essay in segments using the "say something" strategy and also engaged in a number of other activities to help them further understand specific aspects of the text and pursue mathematical questions raised by the reading. For example: students generated their own methods for solving the problem described in the text, how to measure the distance across a lake, before reading the author's; the class "acted out" some of the methods suggested by the author; students engaged in activities to discover and verify the criteria of congruence of triangles prompted by the author's solutions to the lake problem; students read the section on congruence of triangles from their textbook using the "say something" strategy and discussed the connections with their exploration and the essay.

G2. (Egg Man) Recognizing the role played by geometry and mathematical problem solving in the real life construction of a giant egg *(about 2 40-minute class periods)*

In this episode a 20-page essay entitled, "The adventures of an egg man" (Hoffman, 1988), was used to help students appreciate the relevance of geometry to real-life, and more specifically the value of studying the properties of geometric shapes. The students were asked to read this essay and write their questions and comments on index cards as they read, a variation of the "cloning the author" strategy. In the following class the students discussed the reading and their use of the cards, shared their cards in small groups and then a selection were further pursued in both the whole class and a follow-up homework assignment. In the class activities, some sections of the text were also revisited and the students "acted out" some of the geometric constructions described in the text. The students were also asked to generate a list of geometric figures using the essay as a source, in preparation for further activities involving the in-depth study of a selection of geometric figures of the students' choice.

G3. (Area Diagrams) Interpreting area diagrams on a pseudo-Greek manuscript without words *(20 minutes over 2 periods)*

In this episode a 3-page pseudo-Greek manuscript containing "diagrams without words" was used to help students understand the notion of conservation of area and begin to appreciate how some area formulas were derived. The diagrams showed how different figures could be cut and transformed into simpler figures whose area formulas were known. Students were told to imagine that this manuscript contained the secrets for calculating the area of various figures and to try to explain them, first with words and then with algebraic symbols. Students worked in pairs, translating three diagrams of their choice. Since students experienced more difficulty that anticipated the following day the teacher engaged the students in "acting out" one of the translations by actually cutting and pasting figures together.

G4. (Flatland) Creating fictional worlds based on geometric figures *(20-30 minutes over 3 periods)*

In this episode excerpts from *Flatland* (Abbott, 1952) were used to help initiate the construction of a city made of Platonic solids, the concluding experience for this geometry unit. The purpose of this reading was to show students an example of a fictional world created on the basis of mathematical concepts. The first excerpt was read in class using a pair "say something" strategy; another excerpt was assigned for homework without specifying any reading strategy; a third was read aloud by the teacher. In each case the reading was followed by a brief class discussion where students grappled with the idea of different dimensions and discussed the connections between *Flatland* and the construction of their own cities.

G5. (Platonic Solids) Learning about the history, applications and characteristics of Platonic solids *(about 15 minutes over a few periods)*

In this episode a 3-page essay entitled, "Salt and Diamonds" (Giant Golden Book of Mathematics), was used to familiarize students with Platonic solids in preparation for using these solids to construct "fantasy cities". The essay discussed crystals in nature and their connection with Platonic solids, identified some geometric characteristics of these solids and also mentioned the role played by Platonic solids in ancient theories about the universe. Students engaged in a variation of the "using cards" strategy as they read the essay and wrote comments/questions on index cards for homework. In a subsequent class discussion the students used their cards to inform a general class conversation about the essay, to inspire the design of their cities and to make public some of their questions. Later the class also pursued questions regarding "how names of geometric figures came about" and "how Plato tries to use these solids in his theory of the universe" that this reading had raised.

Episodes from a probability unit taught in a suburban high school

P1. (Lady Luck) Becoming aware of the origins of probability *(about 20 minutes over 2 periods)*

This episode was part of the first day of the unit. A 2-page text about the history of probability, "The Birth of Lady Luck" (Weaver, 1963), was used as a way to begin to address the overall goal of the unit - understanding and interpreting probability. The students engaged in a whole class "say something" to read the first two paragraphs of the text and completed the text using the "say something" strategy in pairs. The following day the students each wrote two important things they learned from the reading. After turning in their writings, the students shared their surprise at finding out that probability was discovered/invented and they discussed the connections between "luck" and probability.

P2. (Large Numbers) Understanding the relationship between theoretical and experimental probability *(about 55 minutes over 2 periods)*

At the beginning of the third day of the probability unit a 2-page excerpt on the law of large numbers (Davis, 1961) was used to help students understand the relationship between theoretical and experimental probability and thus continue to deepen their ability to understand and interpret probability. Using a variation of the "using cards" strategy the students took about 8 minutes to read the excerpt and write on a card either a question or something important they felt they had learned from the reading. The teacher collected the cards and then asked the students to explain the law of large numbers in their own words. The class then went on to share and discuss their results from a series of probability "experiments". As they discussed their "experiments" the teacher was able to help the students make the connections between experimental and theoretical probability and understand the role played by the law of large numbers. Several days later, before the unit test, the students revisited the meaning of the law of large numbers by reviewing and responding to the cards they had written.

P3. (News Articles) Interpreting probabilities in real life situations *(about 20 minutes)*

In this episode three current newspaper articles were read using a variation of the "cloning an author" strategy to demonstrate the complexity of interpreting probabilities in real life examples. First, students read the articles for homework with the task of identifying and explaining, in writing, the use of probability made in each. In the following class students first shared and compared what they wrote with a partner, and then discussed them as a whole class. Throughout the class discussion the teacher called attention to the conflicting interpretations being proposed to help students appreciate the complexity of the task and the implications with respect to the notion of probability.

Episodes from a logic unit and a probability unit taught in an urban comprehensive high school

L1. (Nutrition) Developing a controversy around nutrition and diet *(35 minutes over 2 periods)*

This episode took place on the second day of the unit in response to issues students raised during a discussion of diet and nutrition. Three articles, each addressing diet and nutrition in a particular context: "Take Shape" (*Teen*, October 1989), "Nutrition Know-How" (Gibbons, 1976), and "How to Eat your ABC's" (Jones, 1976) were used to stimulate a controversy as students searched for answers to questions about the nutritional value of different foods. Using a variation of the "say something" strategy students formed groups of 2-3, selected an article and read it individually stopping when they found something important to share with their group and record in their notebook. On the following day, after conferring with their group members, spokespersons shared with the whole class what their groups found important. This was followed by a discussion of the categorization of foods based on their current understanding of food groups. The discussion gave the teacher the opportunity to introduce the notion of variable.

p1. (Questions) Developing an understanding of what constitutes a probability question *(1 40-minute class period)*

In this episode a paragraph from a history of probability book (Weaver, 1963) was used to stimulate discussion about the nature of probability questions. This particular paragraph was essentially a list of real life situated questions, for example, "If we raise college tuition by $300 will it discourage so many applicants that we will end up with *less* income from fees ?". In adult facilitated small groups, students read the paragraph using "say something" and discussed issues about the questions in the reading and about writing probability questions. The students subsequently wrote their own probability question on a 3 x 5 card. The teacher later transformed these questions into statements and assigned fictitious probabilities to them to stimulate a discussion of the possible interpretations of the numbers used in probability situations.

Episodes from the "Math Connections" course taught in a public urban alternative high school

C1. (Fig Leaf 1) Learning new strategies for processing and sharing information *(3 1 2 1-hour class periods)*

This episode used sections from an essay, "Underneath the Fig Leaf" (Davis & Hersh, 1981), to introduce some new strategies for making sense of and sharing information gathered from text, as well as and to show historical connections of mathematics to everyday life. [The sections selected: the introduction, were chosen to begin to make historical connections between mathematics and everyday life. In order to learn the "say something" and "sketching" strategies for processing and sharing what one reads, students engaged in a number of instructional activities.] The students first read the essay's introduction as a whole class using the "say something" strategy to learn how it works. This was followed by an explicit reflection on this way of reading. Students then chose, based on their interests, one of the remaining two sections, "Mathematics in the Marketplace" and "Mathematics in War", to read in pairs using the "say something" strategy. After reading the essay, the pairs prepared "sketches" to share *what they learned about mathematics from this piece* with the rest of the class. In an interactive discussion, each pair shared their understanding of the readings by explaining both the content of their "sketch" and the process they used to create it. Finally the whole class engaged in a reflective discussion about the essays in general, how and why they were difficult; and, about the value of the two strategies for processing and sharing information.

C2. (Fig Leaf 2) Learning new strategies for processing and sharing information *(2 1/2 1-hour class periods)*

This episode made use of the same essays as in Part 1, "Mathematics in the Marketplace" and "Mathematics in War" (Davis & Hersh, 1981), to introduce students to another strategy for making sense of and sharing information. Individual students used a variation of the "using cards" strategy to read whichever essay they had not previously read and prepare a set of cards each containing a single idea, question, comment, etc. regarding the essay. The students then used these cards in a variety of ways as they continued to make sense of the essays. For example, the cards prepared on the "Mathematics in the Marketplace" were shared in small groups to settle a controversy raised by the discussion of a sketch previously presented on that essay. Each group selected a card which they felt represented the central idea of the essay and then elaborated their thoughts in a whole class discussion. Each student then created his own "map" of the essay using the class set of central idea cards. During a sharing session students talked about the process and use of the strategy and the content of the essay as they shared their "maps". The students who prepared cards on the "Mathematics and War" essay instead "mapped" their own cards to show the sense each of them had made of the essay. These maps were shared and discussed with the entire class. Throughout these experiences the teacher explicitly pointed out how the cards were acting as springboards to raise questions, make connections, and push for further explanations and the students agreed they were valuable experiences and helpful ways to share information.

Practicing / Implementing the new strategies by using them to teach

In the following episodes (C3 - C7) selected essays from *The Mathematical Experience* (Davis & Hersh, 1981) were read by individual students who then shared what they learned with the whole class. The specific essays were selected to help students become aware of additional connections to mathematics and to deepen their understanding of the nature of mathematics. The essays came from the Mathematical Models: Utility and the Nonanalytical Aspects of Mathematics sections of the book. Students were given a brief description of each essay and then told to choose one and use any (or all) of the newly acquired strategies to read it and prepare to share what they learned with their classmates. These episodes took place over nine class periods. After all of the essays had been shared and discussed students wrote two reflective journals as a springboard to further examine the reading strategies and the collection of essays. The students shared and discussed their responses regarding (a) their evaluation of the reading strategies and their current and future use of these strategies; and (b) the connections between and among the essays and their relationship to the course.

C3. (Math Uses) Reading and sharing interpretations of the essay "Varieties of Mathematical Uses" *(30 minutes)*

In this episode, "Varieties of Mathematical Uses", a 1-page essay making explicit the multiple interpretations of *useful* as it applies to mathematics, was read and "taught" by five students. Each of the five "teachers" used a variation of card "maps" to facilitate sharing what they learned in a small group. For example, one student presented three colorful maps and talked about her understanding by referring to the maps; another read her cards and explained their arrangement which sparked a discussion of her map; another explained her process but in this case the group felt a need to read the essay together paragraph by paragraph.

C4. (Math & Science) Reading and sharing an interpretation of the essay "On the Utility of Mathematics to Other Scientific and Technological Fields" *(30 minutes)*

In this episode, "On the Utility of Mathematics to Other Scientific and Technological Fields", a 2-page essay discussing the difficulty of connecting all applied mathematics directly to the "man on the street", was taught by one student. This student used a variation of the "using cards" strategy to read and facilitate her sharing. In a whole class conversation she read each card aloud and then tried to elaborate on it and respond to questions from the class. At the end there was a brief, spontaneous discussion about how she was using the cards and about some additional ideas for preparing to present.

C5. (Pure vs. Applied) Reading and sharing an interpretation of the essay "Pure vs. Applied Mathematics" *(20 minutes)*

In this episode, "Pure vs. Applied Mathematics", a 2-page essay identifying the values sometimes placed on pure and applied mathematics, was taught by one student. In a whole class conversation this student explained that he had written down important quotes and some questions. As he read his cards he shared the thinking behind them; when there were questions related to the essay he read directly from the essay. Throughout the conversation the teacher seized the opportunity to explicitly discuss some additional strategies for preparing to share, i.e., what should be done if you don't understand something that seems to be a key point in the reading.

C6. (Conscious Math)) Reading and sharing an interpretation of the essay "Conscious and Unconscious Mathematics" *(20 minutes)*

In this episode, "Conscious and Unconscious Mathematics", a 1-page essay pointing out the pervasiveness of the use of mathematics in the universe, was taught by one student. This student used a variation of the "using cards" strategy to read and prepare a set of cards but in a whole class conversation he actually only read one card aloud. He used the card to engage the class by asking what they thought it meant. Instead of reading his other cards he shared examples of times when we are engaging in the doing of conscious and unconscious mathematics. Students spontaneously related the ideas to their previous class inquiry regarding the mathematics of racing.

C7. (Analog & Analytic) Reading and sharing an interpretation of the essay "Analog and Analytical Mathematics" *(7 1-hour class periods)*

In this episode, "Analog and Analytical Mathematics", a 3-page essay identifying some characteristics of analog and analytic approaches to doing mathematics, was the springboard for a 7 day inquiry. Initially, two students read and prepared to teach this text; one student used a variation of the "using cards" strategy and prepared notes and the other used the "sketch-to-stretch" strategy and prepared a sketch. A discussion took place over three consecutive class periods where the students' discussed analytic and analogical approaches to solve several situations/problems. The whole class freqı ently referred to their notes, the sketch, and the examples as they grappled with understanding the distinctions between the approaches. In order to further deepen and enrich students understanding of analog and analytic approaches to mathematics and to see these reading strategies as relevant to learning and doing mathematics, students also engaged in a series of additional activities. For example: students watched a video, The Theorem of Pythagoras (Project Mathematics!, 1989), created a set of cards regarding the use of analytic or analogic approaches in the video, and then created a map of their cards or wrote a journal entry to share what they learned about the approaches from the video; as students shared their maps and journals the teacher created a generalized list of students' ideas and examples on the board; students used this list to identify themselves as someone who generally uses an analog or analytic approach so as to create heterogeneous small groups with the task of creating a theorem and prove it using both an analog and analytic approach. During the creation of the theorem students used a formula table to get information, read and wrote reactions to a segment of an article abou. the mathematical aspects of paper folding (Pappas, 1987, pp. 39) to make connections between paper folding and the task they were asked to do, used math textbooks to find models for writing a theorem, and read, using a group say something, a number of "snippets" about the Pythagorean Theorem to place it in history and provide an example of how to create a text that would tell a story about their theorems; students shared their theorems, proofs and context stories with the whole class; and finally, they engaged in a whole class discussion reflecting on the content and process of the group work.

C8. (Career) Generating questions for independent inquiry *(1 1/2 1-hour class periods)*

In this episode the Careers and Applications sections of two textbooks (Rising et. al., 1985a; Rising et. al., 1985b) were used as catalysts for individuals to generate questions for inquiry to be pursued in an individual project. The teacher first engaged the entire class in an experience to model what and how to create a "thinking question" which needs a solution. Students then read and wrote "thinking questions" for each of the Career and Application sections in one textbook. During a first small group meeting students shared their "thinking questions", a written defense of the question they had chosen to pursue, and a written plan for finding a solution to their question. They received feedback from their peers and the teacher. Students then pursued the inquiry of their choice and later shared their findings.

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TABLES

RLM episodes / LEARNING OPPORTUNITY	G1 LAKE PROBLEM	G2 EGG MAN	G3 AREA DIAGRAMS	G4 FLATLAND	G5 PLATONIC SOLIDS	P1 LADY LUCK	P2 LARGE NUMBERS	P3 NEWS ARTICLES	L1 NUTRITION	p1 QUESTIONS	C1 FIG LEAF 1	C2 FIG LEAF 2	C3 MATH USES	C4 MATH & SCIENCE	C5 PURE vs APPLIED	C6 CONSCIOUS MATH	C7 ANALOG&ANALYTIC	C8 CAREERS
Nature of mathematics: Understand connection of math to life and other fields	•	•			•	•		•	•	•	•	•	•	•	•	•	•	•
Develop a sense of history of math	•	•	•	•	•	•					•	•					•	
Become aware that math is the product of human activity	•	•				•	•				•	•			•	•	•	•
Become aware of humanistic aspects of math	•			•		•	•	•	•		•	•	•	•	•		•	
Understand nature and scope of branches of math		•						•									•	
Understand metamathematical notions		•					•		•								•	
Learn math content: Understand "big ideas" within a math topic	•	•	•	•			•	•		•							•	
Make connections within mathematics	•	•	•				•		•									
Learn with understanding specific concepts/techniques	•	•	•				•		•								•	•
Become aware of new math facts	•	•		•	•	•	•										•	•
Learn about process: Become familiar with math problem solving strategies and approaches	•	•					•		•							•	•	•
Acquire strategies to make sense of texts and learn from texts	•	•	•	•	•	•	•	•		•	•	•	•	•	•	•	•	•
Acquire strategies to share information read					•		•	•	•		•	•	•	•	•	•	•	•
Learn to voice own thinking (opinions, ideas, etc.)	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•
Learn how to learn	•	•									•	•	•	•	•	•	•	•

Table 6.1: Learning opportunities offered by the RLM episodes

What was read	Why it was read	How it was read
"Underneath the Fig Leaf," paragraph introducing a series of brief essays on various connections between mathematics and life.	-To demonstrate the "say something" strategy before trying it in pairs. -To introduce the two articles students would choose and read in pairs. -To help students appreciate the fact that professional mathematicians see mathematics as connected to many aspects of life.	Class read the paragraph silently and then "said something" out loud; these comments were written on newsprint.
Newsprint of student responses, a written record of the introductory "say something" experience.	-To help students value the shift in their comments from frustration with the language to understanding the connection between the text and their class.	The researcher/teacher read and discussed the students' comments aloud while students followed along.
Student choice: "Mathematics and War", a short essay on the relationship between mathematics and war. "Mathematics and the Marketplace", a short essay on the relationship between mathematics and economics.	-To deepen students' understanding of the applications of mathematics to life by exploring the relationship between mathematics and war. -To develop a strategy that could be used for sharing information in the future.	Two groups (Char, Jolea, & Margie; Van & Shellie) read the text silently, stopping at each paragraph to "say something" about what they had just read. Four other pairs of students also used "say something" to read.
Students' sketches	-To value and learn from the different themes students explored while reading their articles. -To value the uses of sketches to share information.	Students and teachers interpreted the drawings as the artists explained their thinking.
"Powerful statements" recorded by the teacher during the sharing of articles.	-To make explicit and value the insights students expressed about the process and content of the experience.	The teacher read it aloud.
Newsprint of students' reflections abut the "say something" and "sketching" strategies.	-To reinforce students' comments.	Silently, individually (no actual evidence of reading).

Table 7.1: Analysis of what, why, and how students read in the "Fig-Leaf 1" (C 1) RLM episode

What was read	Why it was read	How it was read
"Adventures of an Egg Man", a long essay on the construction of a large egg to serve as a monument.	-To help students appreciate how real life situations may involve doing mathematics in a variety of ways. -To help students recognize the properties of geometric figures and their significance. -To learn a strategy that would support students' engagement with a difficult text.	Students wrote comments or questions on 10 3x5 cards as they read the essay at home.
Teacher's directions for group activity with cards, written on board.	-To provide clear directions for group work and a reminder to refer to during group work if needed.	Teacher read directions aloud and commented on them as she did so; students read them silently if they chose.
Index cards written by individual students.	-To support and deepen students' understanding of the text.	Students worked in small groups; read cards aloud, sorted similar cards into piles, selected the most important of these for further discussion.
Directions for homework assignment.	-To understand the homework assignment.	Students read directions silently and individually.
"Adventures of an Egg Man" and course textbook.	-To help students become aware of the names of many shapes within a meaningful context.	Students searched texts selectively to find names of geometric shapes.
Two cards with questions about circles, prepared by students during initial reading experience.	-To value students' questions. -To use students' questions as the motivation for rereading an important section of the essay.	Teacher read questions from student-generated cards on circles aloud to class.
The section of the "Egg Man" essay on constructing a circle, an ellipse, and an egg.	-To help students understand the construction of a circle and ellipse and relate each to the basic property of that figure. -To provide a demonstration for how to approach the homework. -To answer the student questions that initiated this reading event. -To help students understand similarities/differences between circles, ellipses, and eggs.	Students first read relevant section of text individually and silently; the teacher then guided a rereading of it in which a student actually did what was described in the text for circle and ellipse ("enacting the text"), and addressed at the same time the questions that motivated the reading.
List of "most interesting" index cards (as defined by either students or teacher) prepared by teacher.	--To help students explore a math question in more depth and thus deepen and go beyond their understanding of the essay. --To value the students' ideas and questions about the essay.	Students read through list, chose one question, and tried to answer it for homework.

Table 7.2: Analysis of what, why, and how students read in the "Egg-Man" (G2) RLM episode

RLM episode	WHAT was read Format / content	WHY it was read **To learn about...**	HOW it was read & used
LAKE PROBLEM **G1** **6 40-min. periods**	Essay / origins of geometry textbook / congruence of triangles	geometry origins; geometry/problem solving; congruence of triangles congruence of triangles	Text split in 3 parts; class & pair say something + enact (acting out, fill in gaps, work prior + catalyst for other activities) Homework; say something
EGG MAN **G2** **2 40-min. periods**	Long essay / building of a giant egg	real-life uses of math; geometric figures	Using card + enact (acting out, fill in gaps + catalyst for other activities)
AREA DIAGRAMS **G3** **20 min.**	Hand-out with diagrams without words / area	area formulas and their derivation; interpreting diagrams	Translate + enact (acting out)
FLATLAND **G4** **20-30 min.**	3 sections of novel / *Flatland*	fictional worlds based on math; use of math in literature	1. Say something; 2. Homework + discussion 3. Read aloud + discussion
PLATONIC SOLID **G5** **15 min.**	Essay / Platonic solids	Platonic solids' history and applications (as inspiration)	Using cards
LADY LUCK **P1** **20 min.**	Excerpt from essay / historical context of probability theory	historical & human aspects of probability; reading strategy	Class & pair say something + using cards
LARGE NUMBERS **P2** **55 min.**	Excerpt from essay / how theory of probability applies to averages	experimental vs. theoretical probability;	Using cards + enact (catalyst for other activities)
NEWS ARTICLES **P3** **20 min.**	3 news articles / probability in real life	interpreting real life probability issues	Using cards
NUTRITION **L1** **35 min.**	Choice of 3 articles / nutrition	real life uses for logic tools; nutrition issues (to spark a controversy)	Using cards
QUESTIONS **p1** **40 min.**	Paragraph of questions / probability issues implicit	real life probability questions	Small group say something + using cards + enact (model)

Table 7.3: Summary of what, why and how students read generatively in all RLM episodes

FIG LEAF 1 **C1** **3.5 1-hour periods**	Essays / 1. Introduction 2. Mathematics and War / Mathematics and the Marketplace	strategies for making sense of and sharing information; historical connections between math and everyday life	1. Class say something; 2. Say something + sketch
FIG LEAF 2 **C2** **2.5 1-hour periods**	Essays / 1. Mathematics and the Marketplace 2. Mathematics and War	strategies for making sense of and sharing information	Using cards 1. Maps of selected cards 2. Maps of own cards
MATH USES **C3** **30 min.**	Short essay/ examples & interpretations of "useful" math	implementing a strategy for sharing information: usefulness of math	Using cards (maps) + say something
MATH & SCIENCE **C4** **30 min.**	Short essay/ difficulties connecting applied math to common man	implementing a strategy for sharing information; roles of applied math	Using cards
PURE vs APPLIED **C5** **20 min.**	Short essay/ values placed on pure and applied math	implementing a strategy for sharing information; value issues in math	Using cards
CONSCIOUS MATH **C6** **20 min.**	Short essay / description of levels of awareness of doing mathematics	implementing a strategy for sharing information; pervasiveness of math	Using cards
ANALOG & ANALYTIC **C7** **7 1-hour periods**	Short essay / characteristics of analog & analytic approaches to doing math Directions / mathematical aspects of paper folding Several "snippets"/ historical context of use & proofs of Pythagorean Theorem	implementing a strategy for sharing information; alternative approaches to doing math other analog approaches (to spark ideas) historical context of Pythagorean Theorem (to act as a model)	Sketch, using cards & enact (catalyst for other activity) Reflective journal Class say something; enact (model-create a snippet)
CAREERS **C8** **1.5 1-hour periods**	Several brief sections from textbooks/ careers and applications of math	careers and applications of math (as a catalyst for generating inquiry)	Generate "thinking" questions and pursue one

Table 7.3: Summary of what, why and how students read generatively in all RLM episodes

RLM episodes **VARIATIONS OF TRANSACTIONAL READING STRATEGIES USED:**	G 1	G 2	G 3	G 4	G 5	P 1	P 2	P 3	L 1	p 1	C 1	C 2	C 3	C 4	C 5	C 6	C 7	C 8
Say something: • in pairs	•			•		•					•							
• in small groups										•			•					
• as the whole class	•										•						•	
Ways of sharing: • class discussion	•			•		•					•						•	
Sketch-to-stretch: • done as the text is read for the first time																	•	
• done after the text had already been read											•							
Ways of sharing: • author explains sketch											•						•	
Using Cards: *Ways of writing the cards:* • using multiple cards		•			•							•	•	•	•	•		
• using a single card							•			•								
• writing on notebook rather than cards						•		•	•								•	
Ways of sharing/using the cards: • cards read by teacher as source of information		•			•	•	•			•								
• cards shared in class discussion		•			•			•	•			•		•	•	•	•	
• cards shared in small groups		•			•		•	•	•	•		•	•					
• cards are categorized and/or selected		•					•		•			•				•		
• own cards are mapped												•	•					
• class selection of cards is mapped												•						
• some cards are pursued further		•					•	•		•			•	•	•	•		
Enacting: • "acting out" what described in the text	•	•	•															
• using text as model										•							•	
• "fill-in gaps"	•	•																
• working out solutions prior to reading	•																	
• use text as catalyst for other math activity	•	•					•										•	
Ways of sharing: • done as a whole class	•	•	•							•								
• follow-up class discussion																	•	
Others: • generate thinking questions and pursue one																		•
• translate nonverbal text / symbols			•															
Ways of sharing: • done as a whole class																		
• follow-up class discussion			•															•

Table 7.4: Variations of transactional reading strategies used in the RLM episodes

RLM episodes / WAYS TEACHER VALUED THE GENERATIVE READING EXPERIENCES:	G1 LAKE PROBLEM	G2 EGG MAN	G3 AREA DIAGRAMS	G4 FLATLAND	G5 PLATONIC SOLIDS	P1 LADY LUCK	P2 LARGE NUMBERS	P3 NEWS ARTICLES	L1 NUTRITION	p1 QUESTIONS	C1 FIG LEAF 1	C2 FIG LEAF 2	C3 MATH USES	C4 MATH & SCIENCE	C5 PURE vs APPLIED	C6 CONSCIOUS MATH	C7 ANALOG & ANALYTIC	C8 CAREERS
• teacher articulation of rationale and value of the strategy when first introduced	•	•		NA	NA						•	•	•	NA	NA	NA	•	•
• teacher articulation of rationale and value of the reading experience students is engaging in	•	•		•	•	•		•			•	•	•	•	•	•	•	•
• significant class time is devoted to reading and/or follow-up activities	•	•	•	•	•				•	•	•	•	•	•	•	•	•	•
• significant class time is devoted to "sharing" what students made of the reading	•	•		•	•		•				•	•	•	•	•	•	•	•
• explicit class discussion on the process and/or value of the reading experience	•	•									•	•	•	•	•	•	•	•
• students reflect on the process and its value in a journal entry or other written assignment	•												•	•	•	•	•	•
• students' spontaneous comments on the process are encouraged and capitalized upon	•	•		•							•	•	•	•	•	•	•	•

Table 8.1: Ways teachers valued the generative reading experiences

WAYS TEACHERS INTRODUCED NEW READING STRATEGIES: / RLM episodes	G1 SAY SOMETHING	G1 ENACT TEXT	G1 ENACT CATALYST	G2 CARDS	G3 TRANSLATE	P1 SAY SOMETHING	P2 CARDS A*	P3 CARDS B*	L1 CARDS	p1 SAY SOMETHING	C1 SAY SOMETHING	C1 SKETCHING	C2 CARDS CLASSMAP	C2 CARDS OWN MAP	C7 ENACT CATALYST	C7 ENACT MODEL	C8 QUESTIONS
Making explicit the scope of the strategy: • teacher articulates rationale and value of the strategy	•			•							•	•	•	•	•	•	•
• explicit class discussion on the process and/or value of the strategy after it is experienced	•			•							•	•	•	•	•	•	•
Providing an image of how the strategy could play out: • teacher models the strategy (with students' participation)	•	•			•	•				•	•			•			•
• teacher gives very detailed instructions about what the students are expected to do				•	•		•	•					•			•	
• teacher makes connections to other known strategies to explain the new one				•		•				•	•		•	•			

Note: In episodes P1 and P2 A and B are used to show that each episode used a different variations of the card strategy.

Table 8.2: Ways teachers introduced new reading strategies

FIGURES

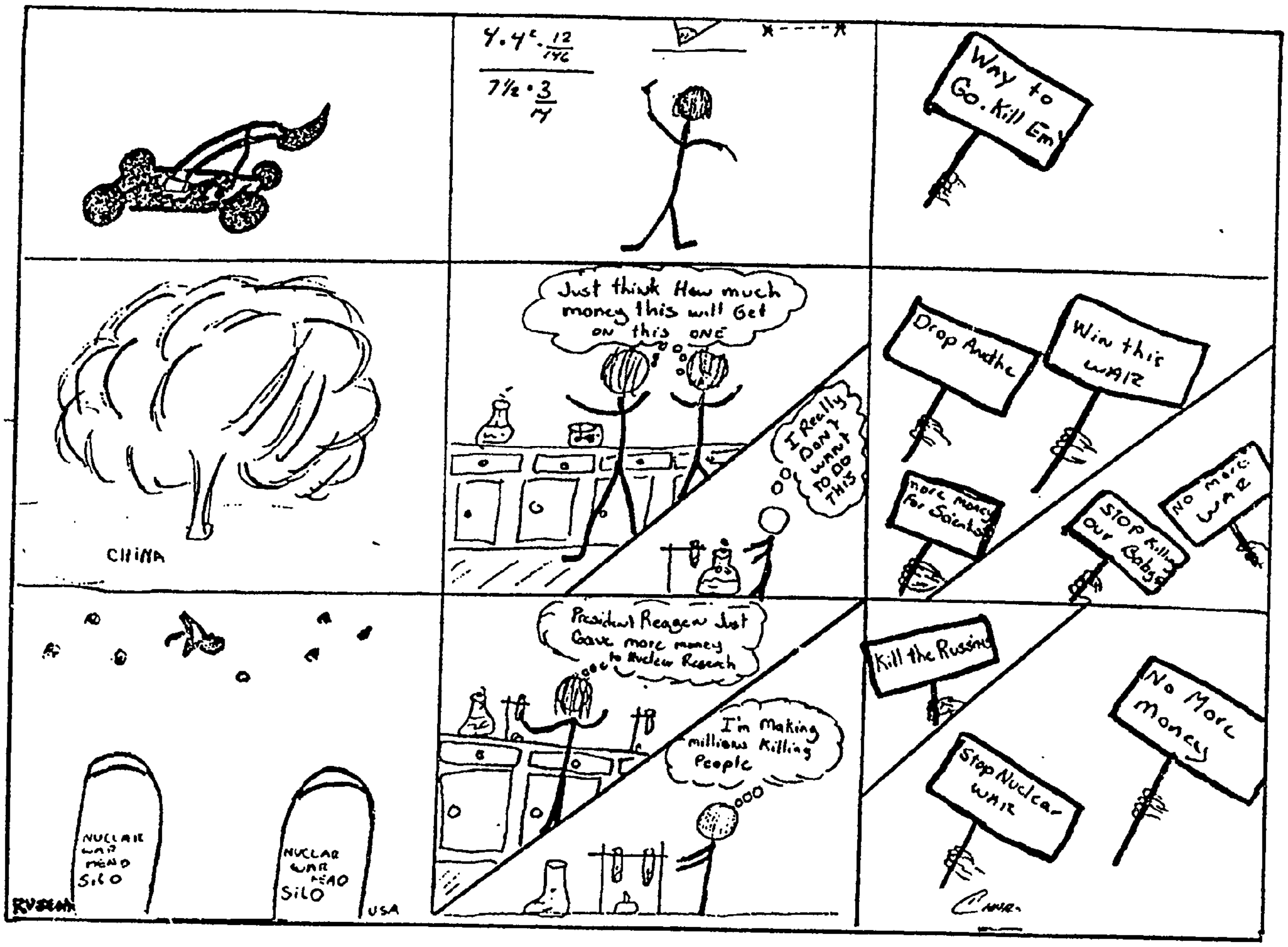

Figure 4.1: Char's sketch of "Mathematics and War"

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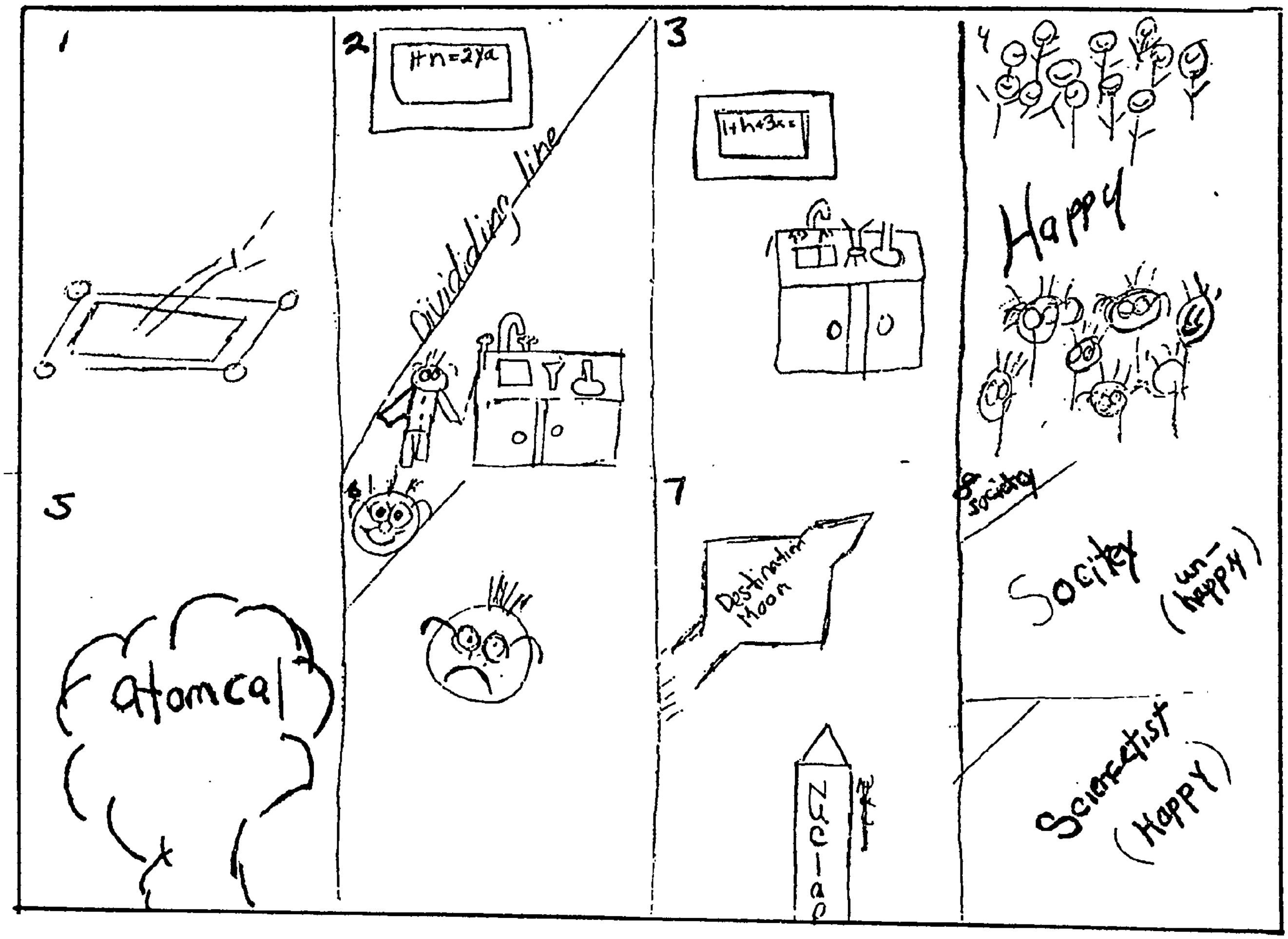

Figure 4.2: Jolea's sketch of "Mathematics and War"

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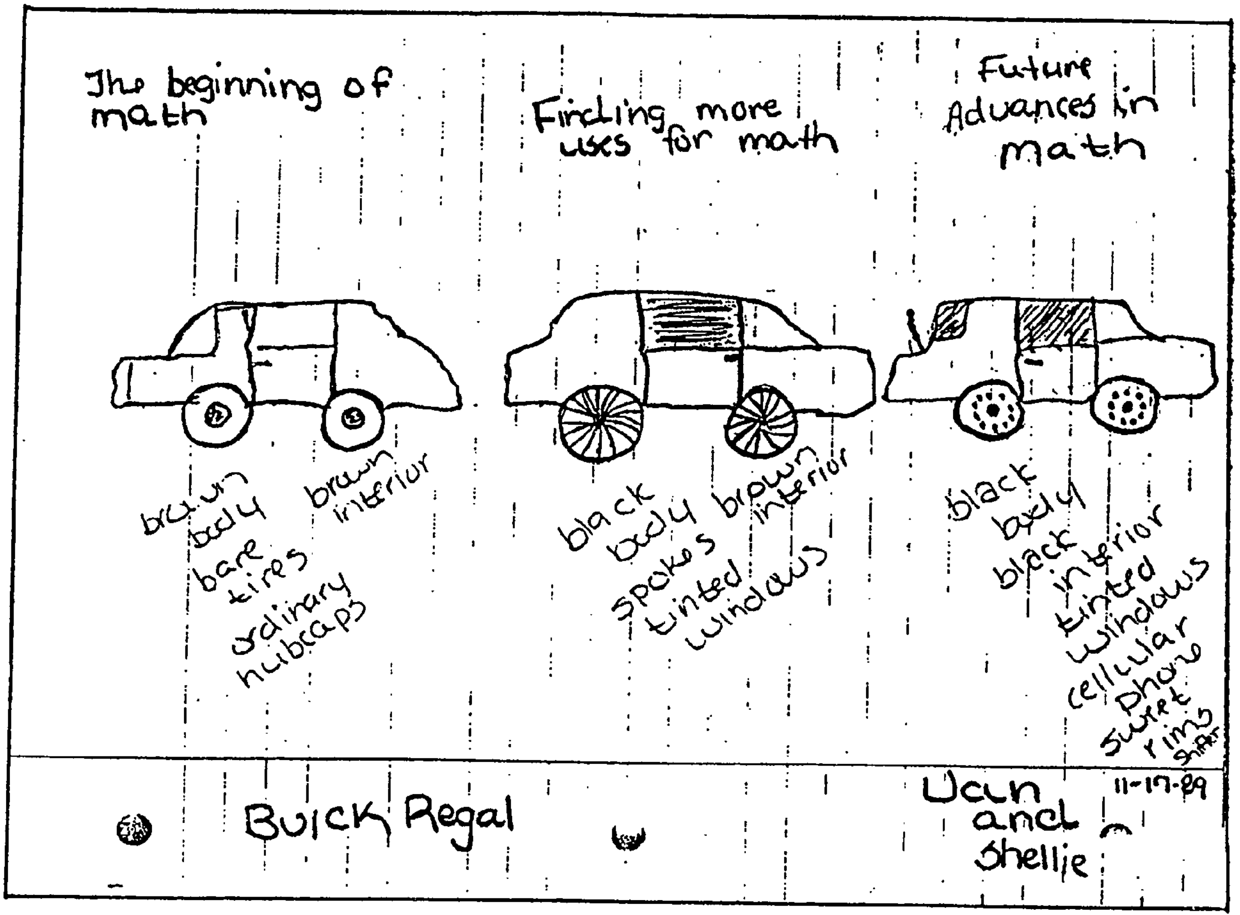

Figure 4.3: Van and Shellie's sketch of "Mathematics and War"

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